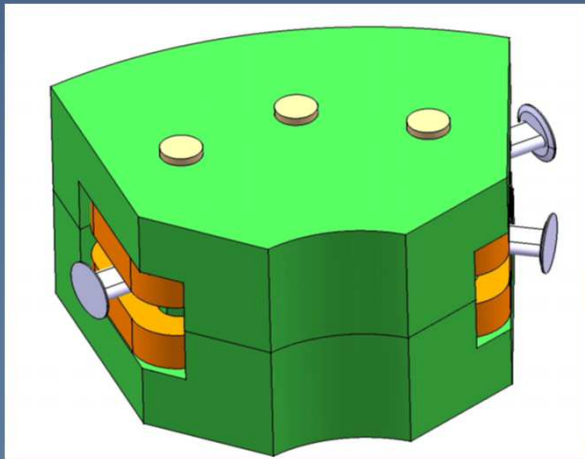
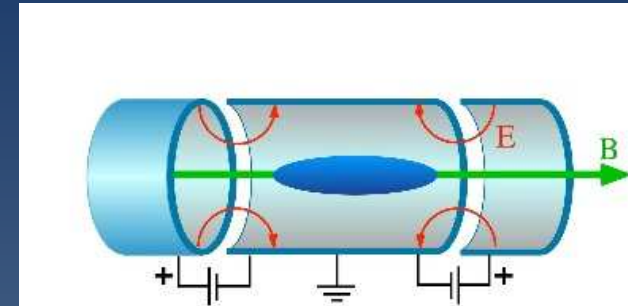
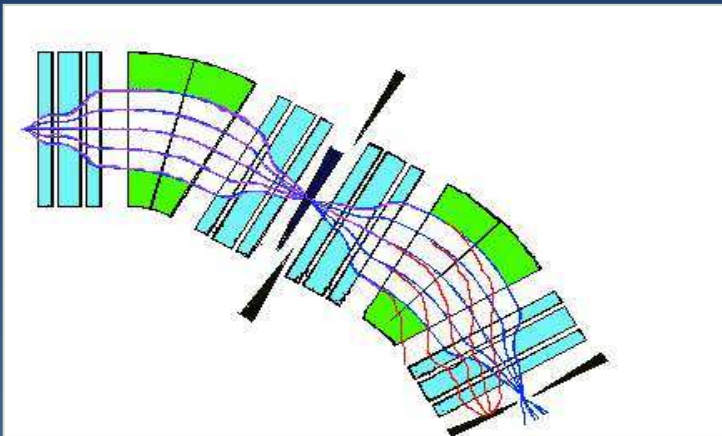


Electromagnetic Spectrometers



Summary

- I) History and evolution of the spectrometers
- II) Magnetics spectro/separators with accelerator's beams
technical devices : quads, dipoles
Beam optics concept
- **III) Spectrometers without accelerator**
 - 1 exemple for Astroparticle
 - Penning Traps

I) History/evolution of the electromagnetic spectrometers

1) **Thomson (1897)**: cathode rays measurement

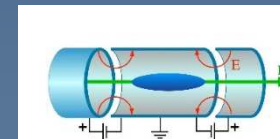
Noble prize in 1906 for discovery of the electron (measurement of m_e/q_e ratio with **E+ B selection**)

2) **Aston (1919)** : **E+ B selection**

identification of isotopes Ne,Cl & mass measurement

3) **40's** :**Manhattan project** U235/U238 enrichment (**B selection**)

4) **Dehmelt (1955)** :**Penning Traps**



5) « **Commercial** » **mass analyzer** : (Tof ,Maldi,Esi,...)

Small size tools commercially available for many applications

....

Magnetic selection : the basic things

Ion Equations in a transverse magnetic field

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} = q (\mathbf{v} \times \mathbf{B})$$

Lorentz force : $(\mathbf{v} \perp \mathbf{F})$ so $\mathbf{v} \cdot d\mathbf{v}/dt = dv^2/dt = 0$
hence the modulus $|\mathbf{v}| = \text{Constant}$ and $\gamma = \text{constant}$
The motion is circular and uniform:

$$\frac{d\mathbf{v}}{dt} = \frac{|\mathbf{v}|^2}{R} e_r$$

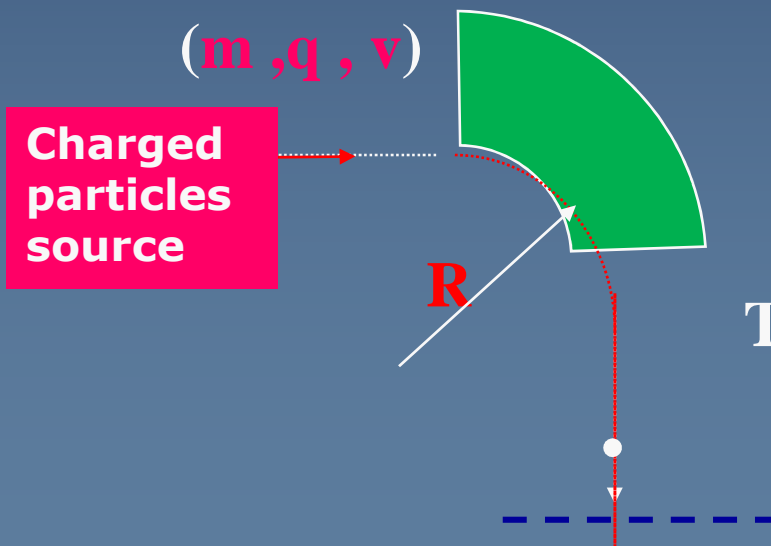
$$\gamma m v^2 / R = q |\mathbf{v}| |\mathbf{B}|$$

trajectory Radius **R**

$$R = \gamma \frac{mv}{qB}$$

We define the particle rigidity : $B\rho$ [Tesla.m]

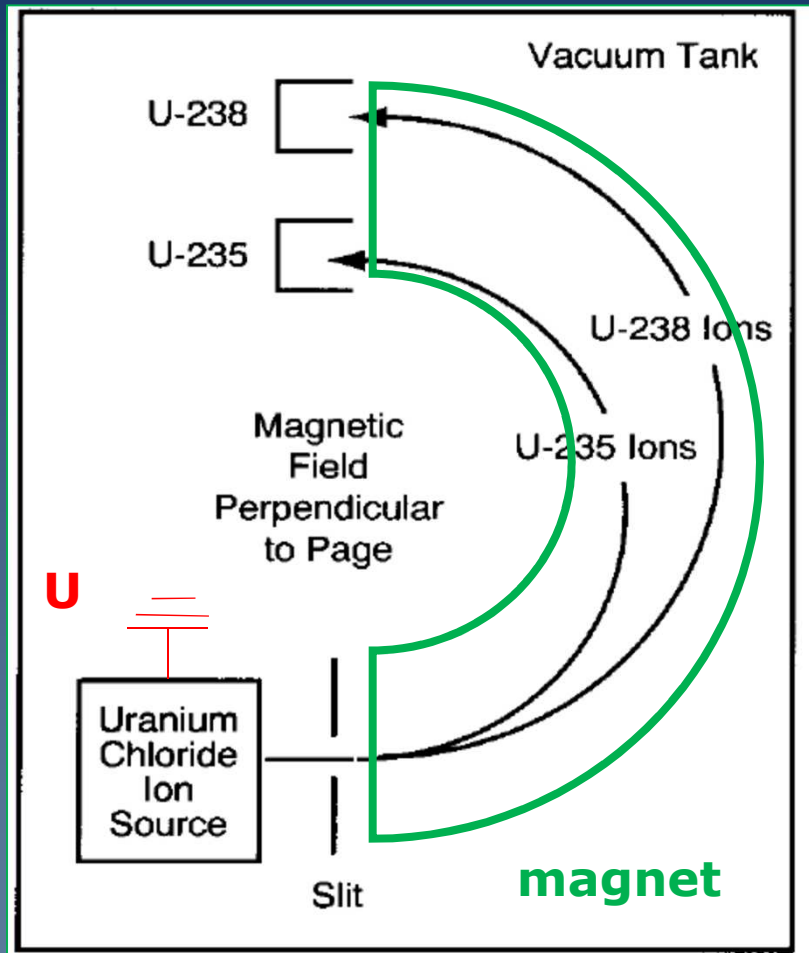
$$B\rho \stackrel{\text{def}}{=} \gamma \frac{mv}{q}$$



The trajectory radius given By $R = B\rho / B$

$$R = \frac{B\rho}{B} = \gamma \frac{mv}{qB}$$

Isotope separator for atomic bomb (1942)

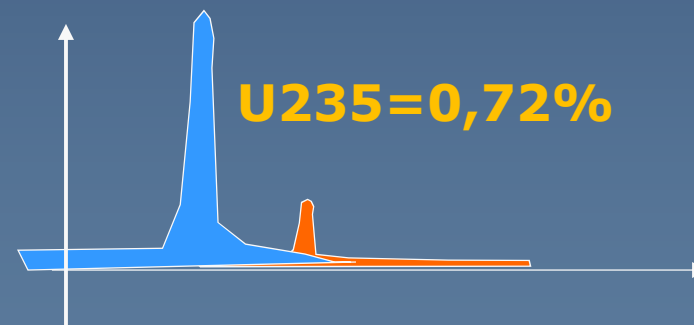


$$R = \frac{B\rho}{B} = \gamma \frac{mv}{qB}$$

$$V = (2qU/m)^{1/2}$$

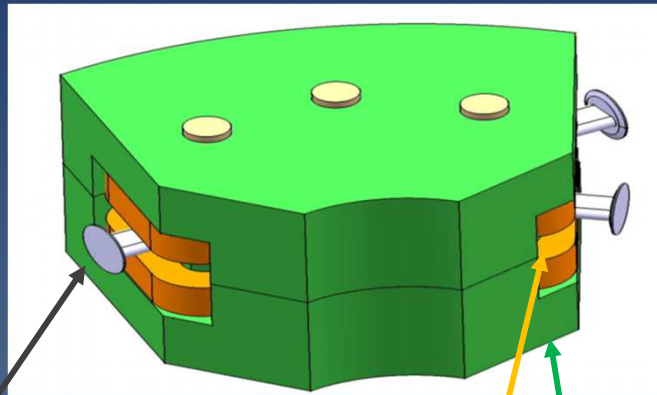
$$R_{235}/R_{238} = (235/238)^{1/2}$$

U238 : 99,27%



1152 isotope **magnetic separators** at Oak Ridge (USA) in The 1940's for ^{235}U bomb

Magnetic dipole : technical details



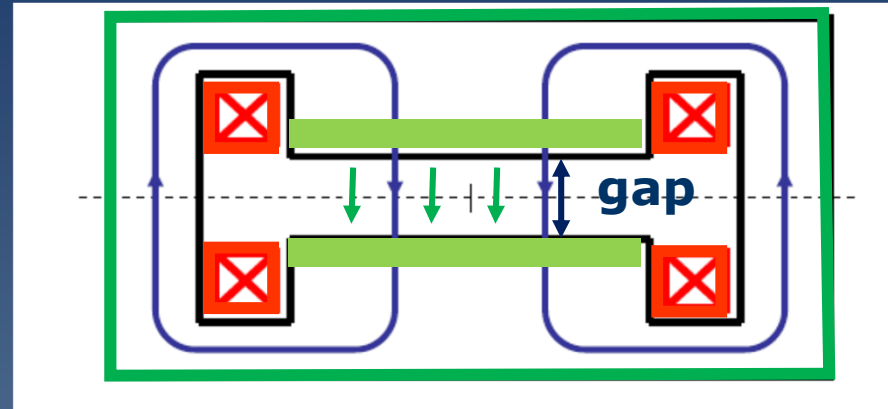
Beam pipe (vacuum)
Coils (copper)

Yoke & Poles (N & S)

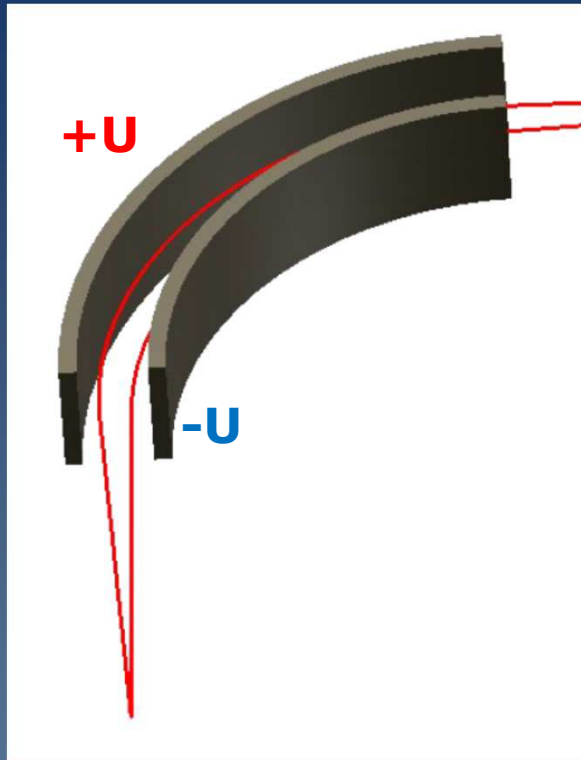
Poles saturates at 1,6-2Tesla
 $B_z \sim I_{\text{power supply}} / \text{gap}$

Power supply (100-1000A)

H type magnetic dipole



Electrostatic selection* :

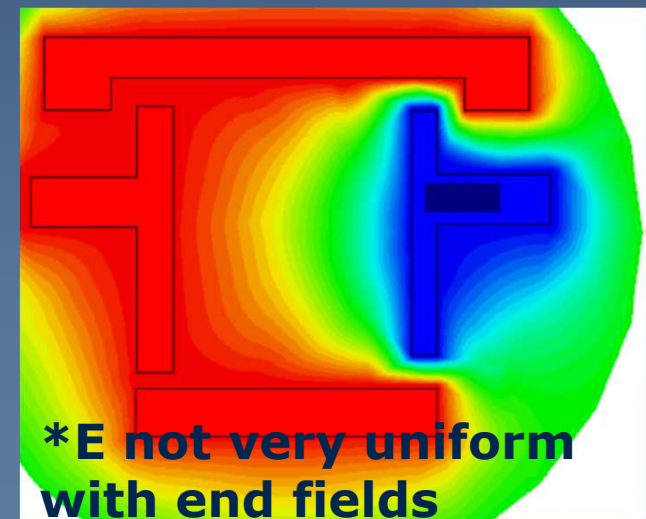
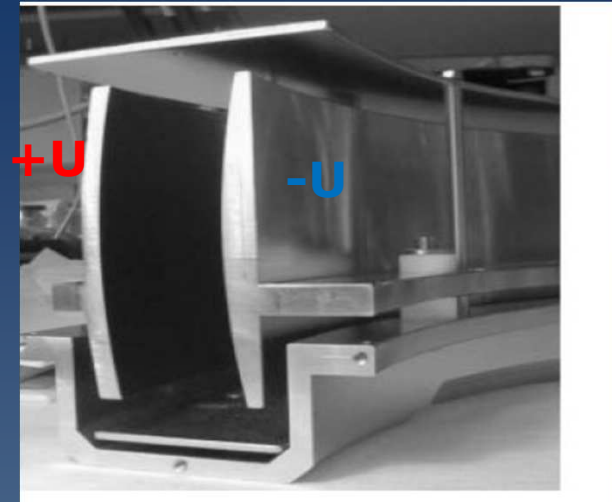


$$F = q E$$

$$R = \gamma m v^2 / q E$$

*Difficult to bend energetic particle
With
raisonnable E.field

*sparking



*E not very uniform
with end fields

E not very efficient

Electric force $\sim q E$
low energy particles keV ion/electron

Magnetic force $\sim q v B$
adapted to high energy particle

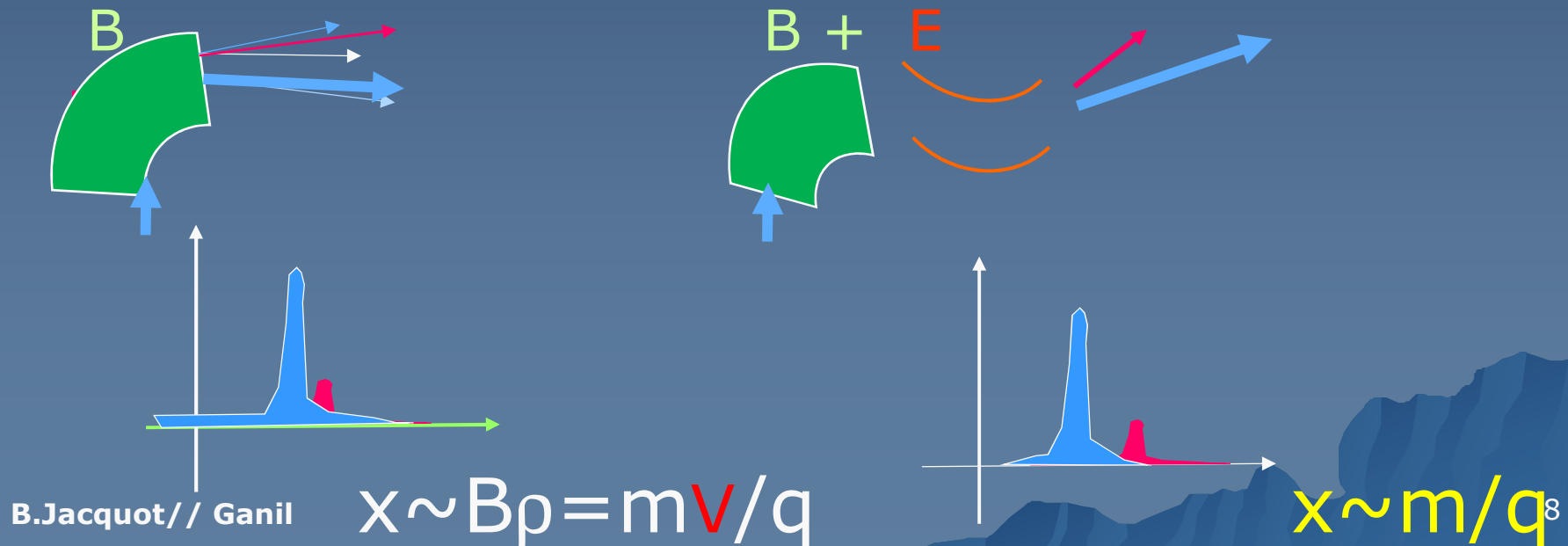
Electrostatic selection : many limitations but ...

- Lower weight/cost than electromagnet
- **Electric deflector** can complement **Magnet**
(velocity compensation possible $R_E/R_B \sim v$)

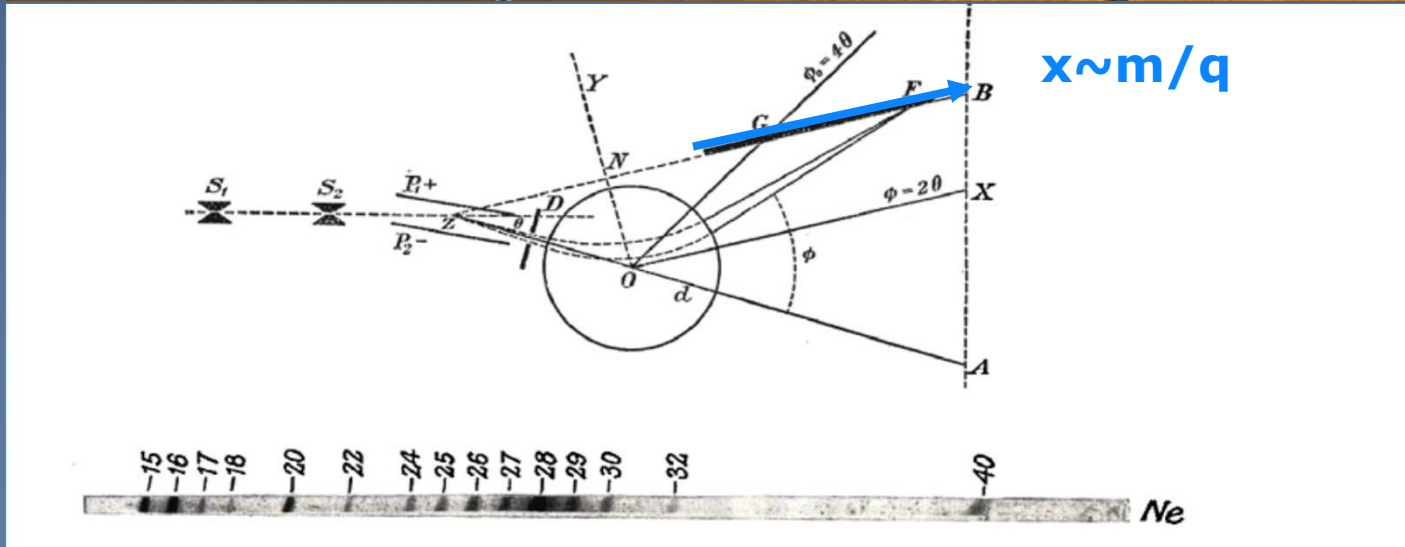
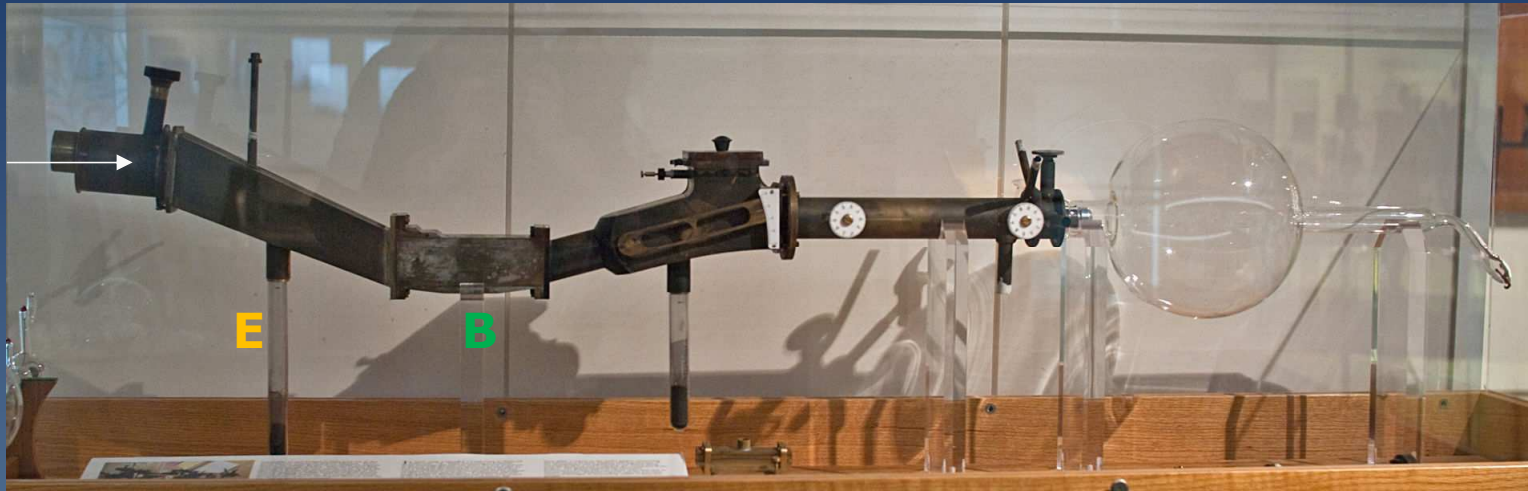
F.W. Aston : Nobel prize in 1929 with a

« **mass spectrograph** »

identification of ^{20}Ne ^{22}Ne



F.W Aston Nobel Prize : Stable isotopes discovery : $^{20-22}\text{Ne}$; $^{35-37}\text{Cl}$

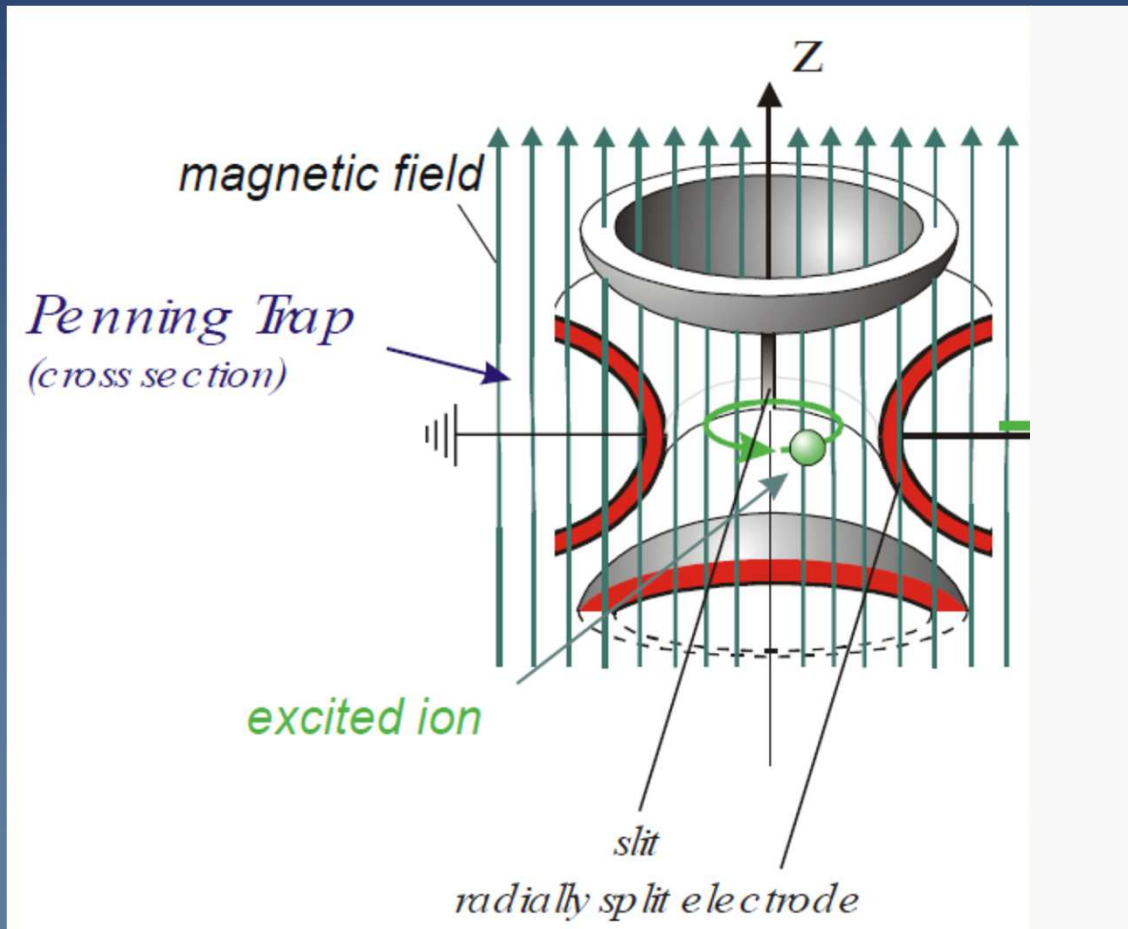


Penning Traps : high resolution mass spectroscopy

$R_m \sim 10^5 - 10^9$

Developped in the 50s (**Dehlmert**)

Used in research but the applications are expanding



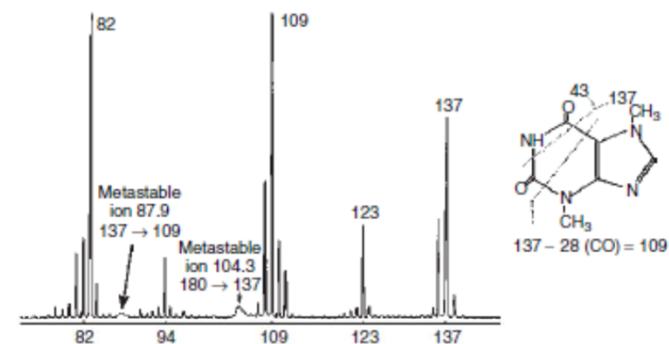
3 tricks

- 1) Confinement ($B_z + E_z$)
static EM fields
- 2) Excitation (RF field)
- 3) Extraction
(Tof measurement)

Commercial Mass analysers

TOF or magnetic & electrostatic deflection
Chemistry, Pharmacy, industry

Importance of electromagnetic spectrometers



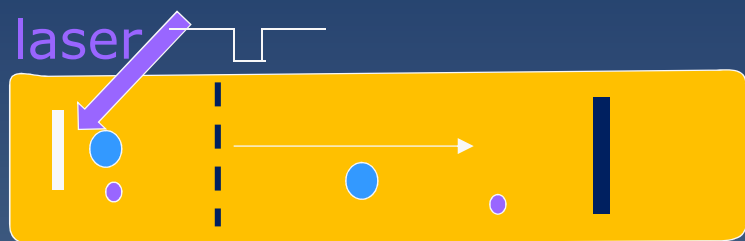
Molecule identification

Accurate mass measurements

- Quantitation
- Isotope ratio measurements

« Commercial » tools : Small ToF spectrometer for mass analysis

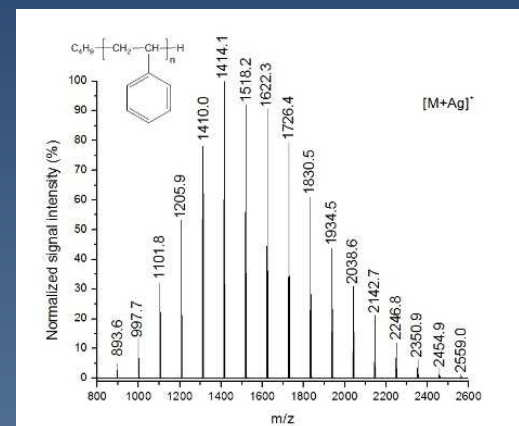
Time of flight : ionisation (pulsed laser) + acceleration



+U 0V Time detector

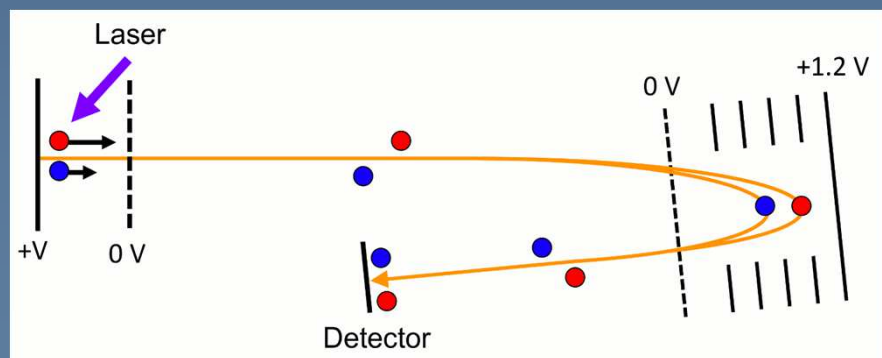
$$qU = mV^2/2 = m (L/Tof)^2$$

$$Tof = L (m/qU)^{1/2}$$



Reflectron (Tof ~ m/q)

: isochronism (Tof non dependant of U)



$$Tof = L (m/q)^{1/2}$$

$$L = F(U)$$

for $U = U_0$

$$L = L_0$$

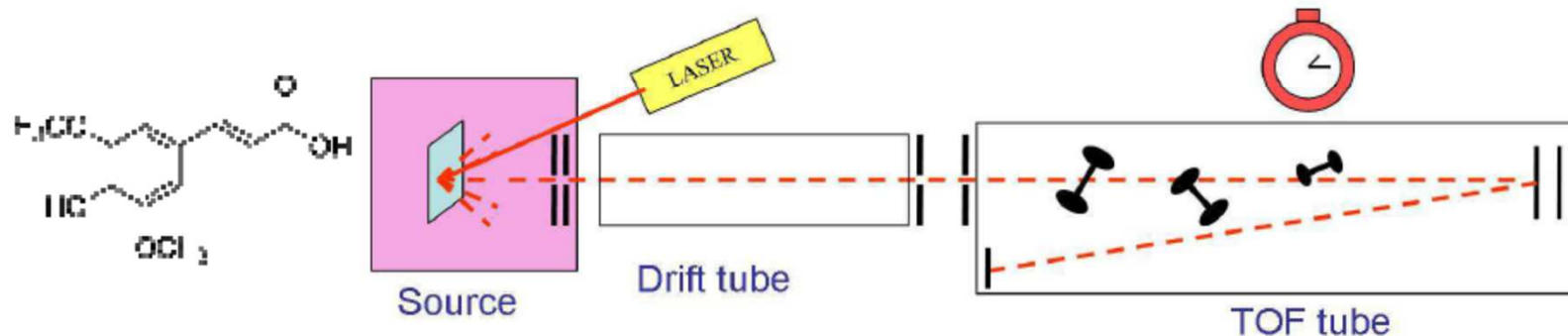
for $U = U_0 + \Delta U$

$$L = L_0 + \Delta L$$

Commercial tools for Mass spectroscopy

Maldi = **M**atrix **A**ssited **L**ASER **d**esorption / **I**onization

MALDI-TOF matrix assisted laser desorption/ionization TOF

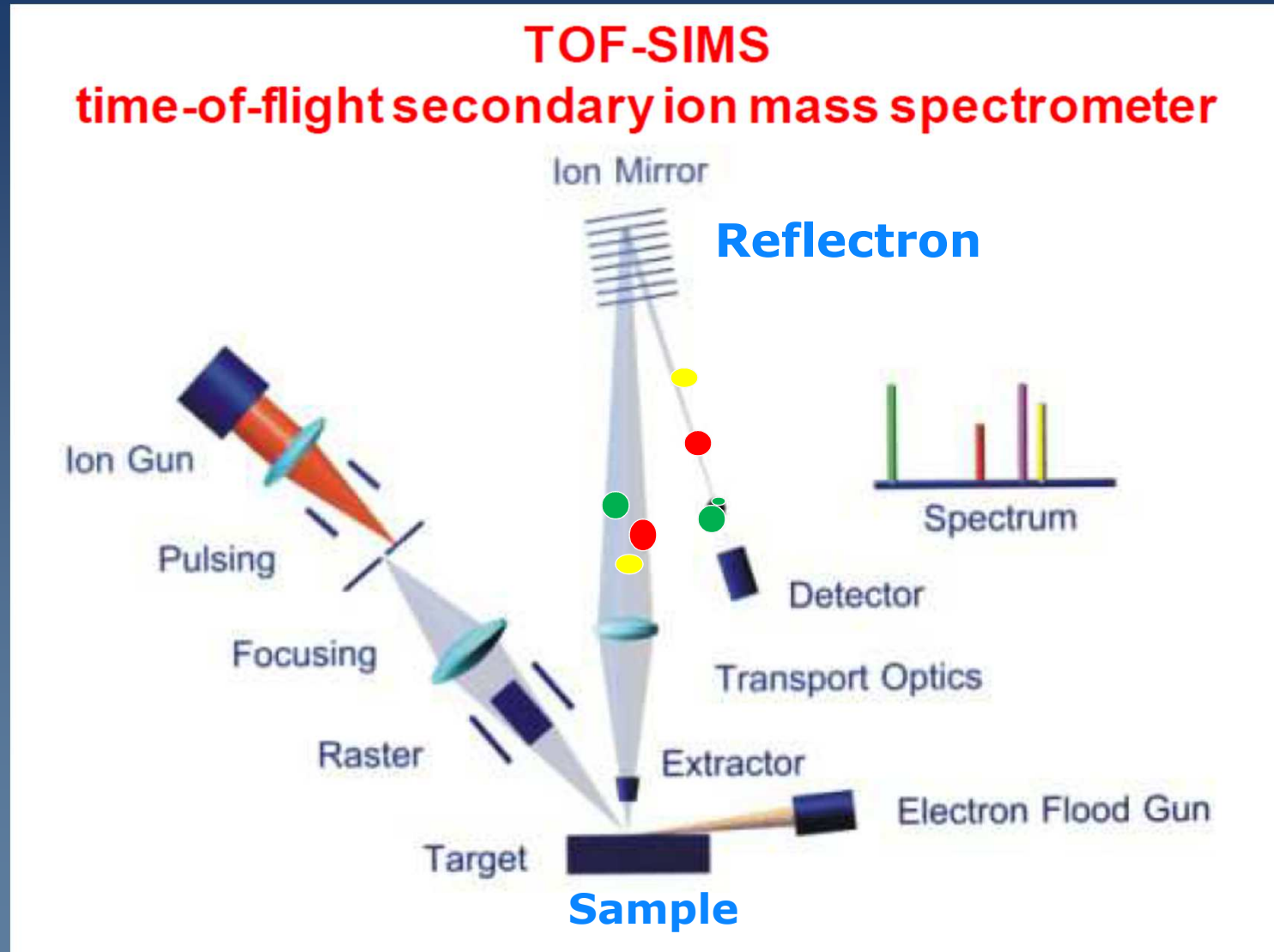


Matrix:

- low vapor pressure for operation at low pressure
- polar groups for use in aqueous solutions
- strong absorption in UV or IR for efficient evaporation by laser
- low molecular weight for easy evaporation
- acidic: provides easily protons for ionization of analyte

Commercial tools for Mass spectroscopy

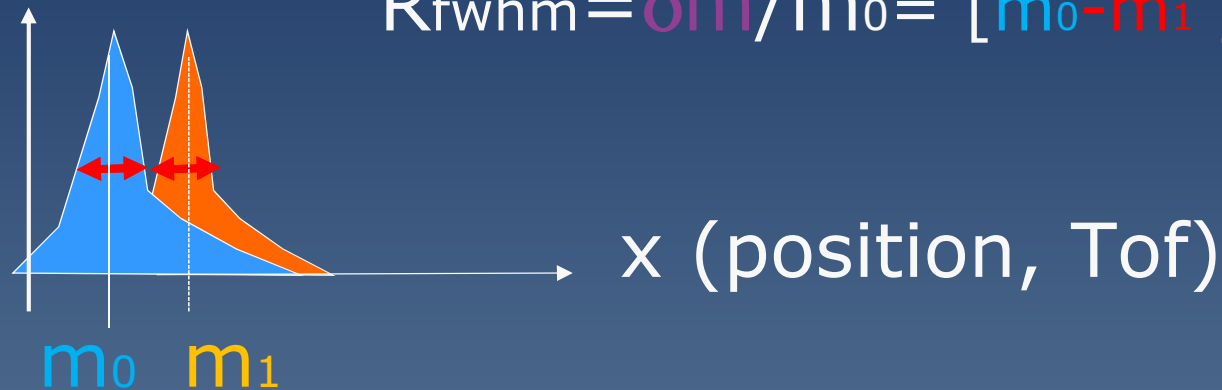
SIMS = ionzation with an ion gun of a sample



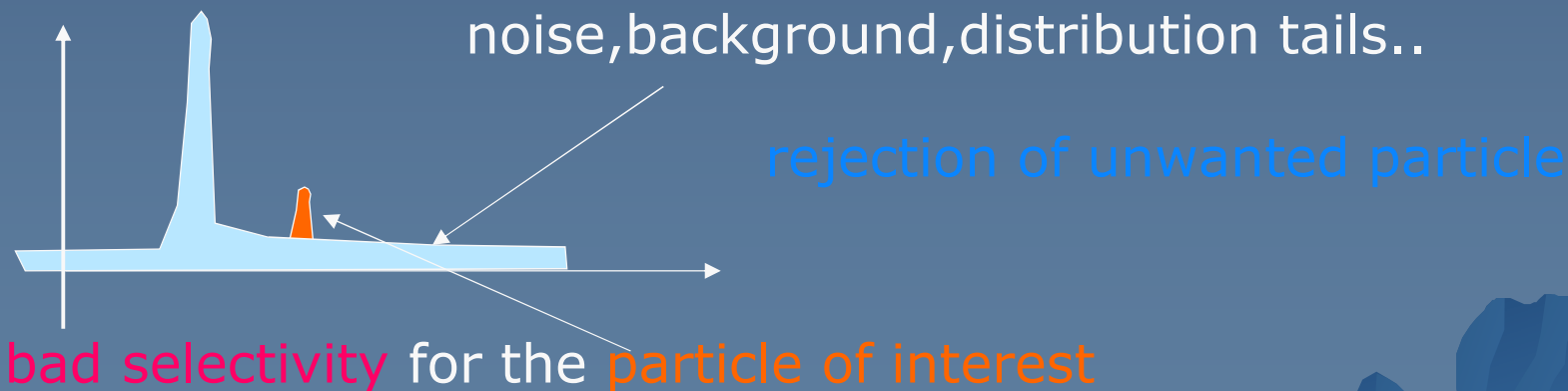
Spectrometer measurement : what are the performances ?

Resolution (closest peak separation)

$$R_{\text{fwhm}} = \delta m / m_0 = [m_0 - m_1] / m_0$$

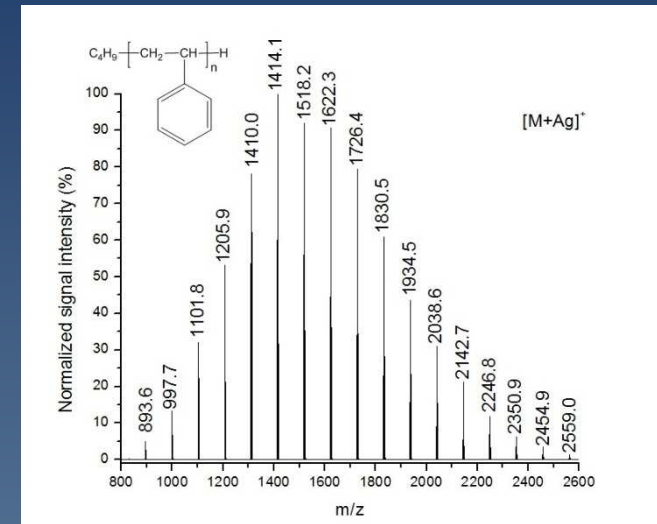
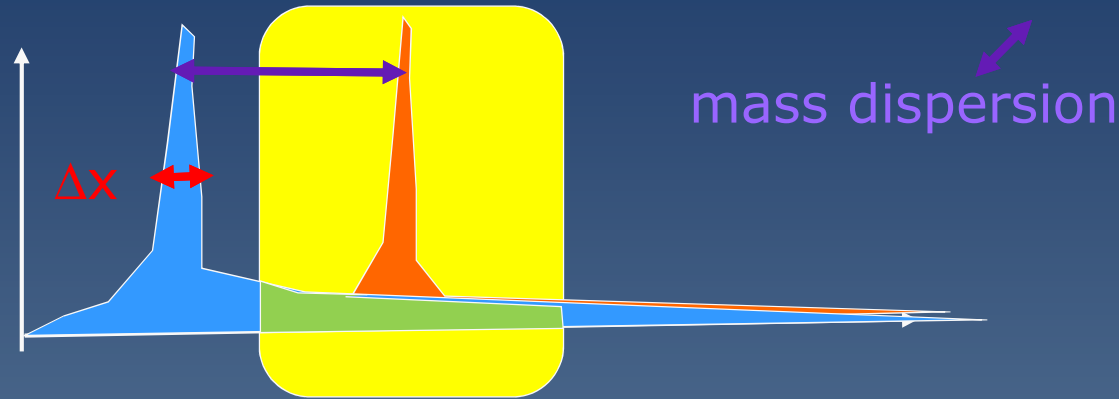


Selectivity (clean or not clean)



Spectrometer measurement : performances

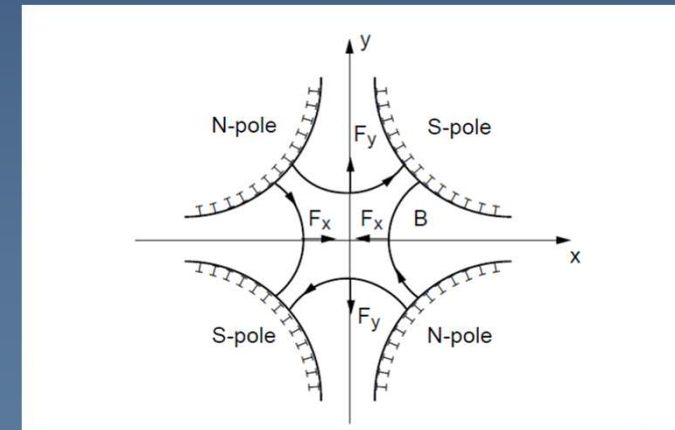
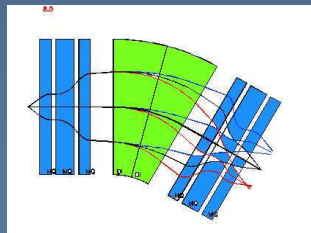
-Resolution $R_{fwhm} = \Delta x_{fwhm} / dx/dm$



-Acceptance : -Mass range
 -Angular acceptance
 (solid angle: steradian)

Rejection = Nb of unwanted particles (final)
 / Nb of unwanted particles (initial)

II) Spectrometers & separators with accelerator's beams



$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_2 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1$$

$$l = v_0(t - t_0)$$

$$\delta = \frac{B\rho - B\rho_0}{B\rho_0}$$

Spectrometers & separators with accelerator beams

1) Why a spectrometer ? Part 1

2) Designing a spectrometer (1st approach)

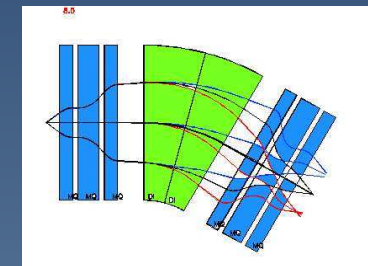
3) Beam optics (Basics)

4) Spectrometer's **properties**

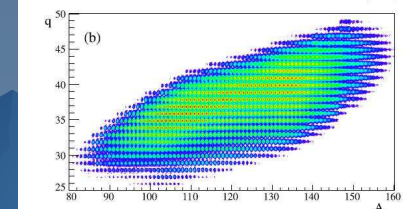
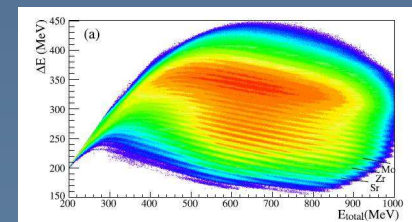
Part 2

5) Fragments separators (100MeV/A-500 MeV/A)

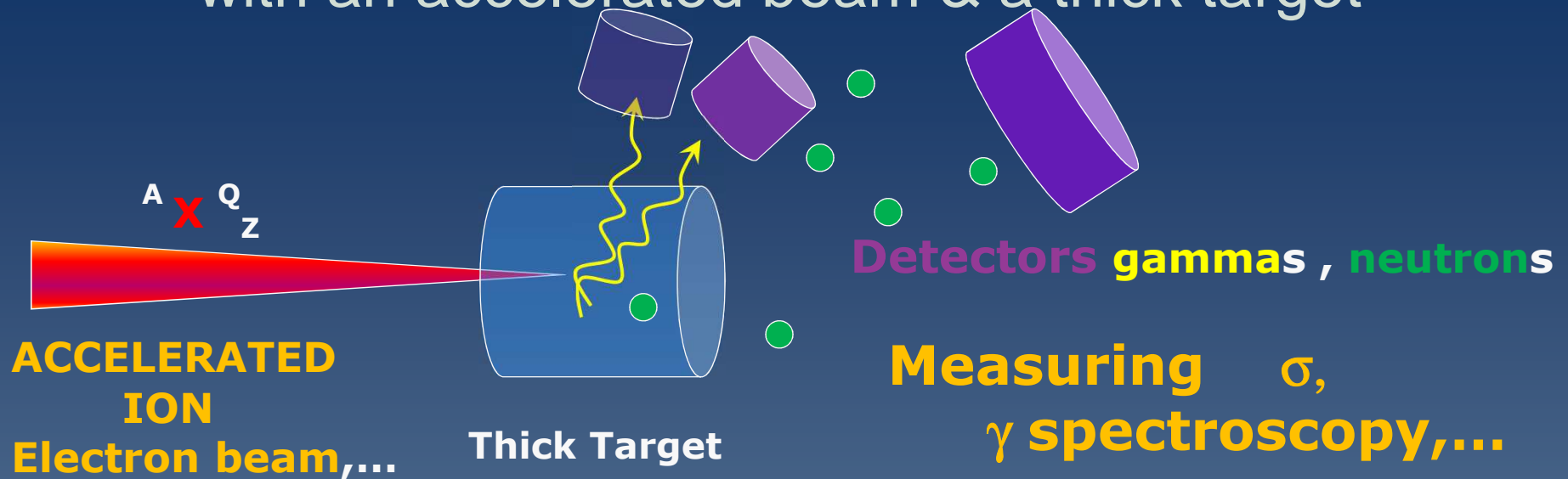
6) Tuning And Diagnostics



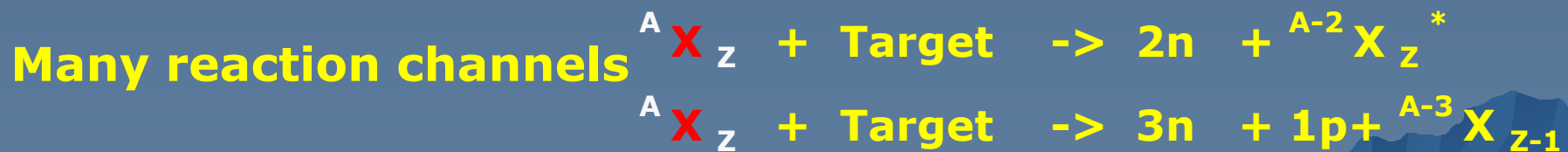
$$\begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



An experiment with an accelerated beam & a thick target

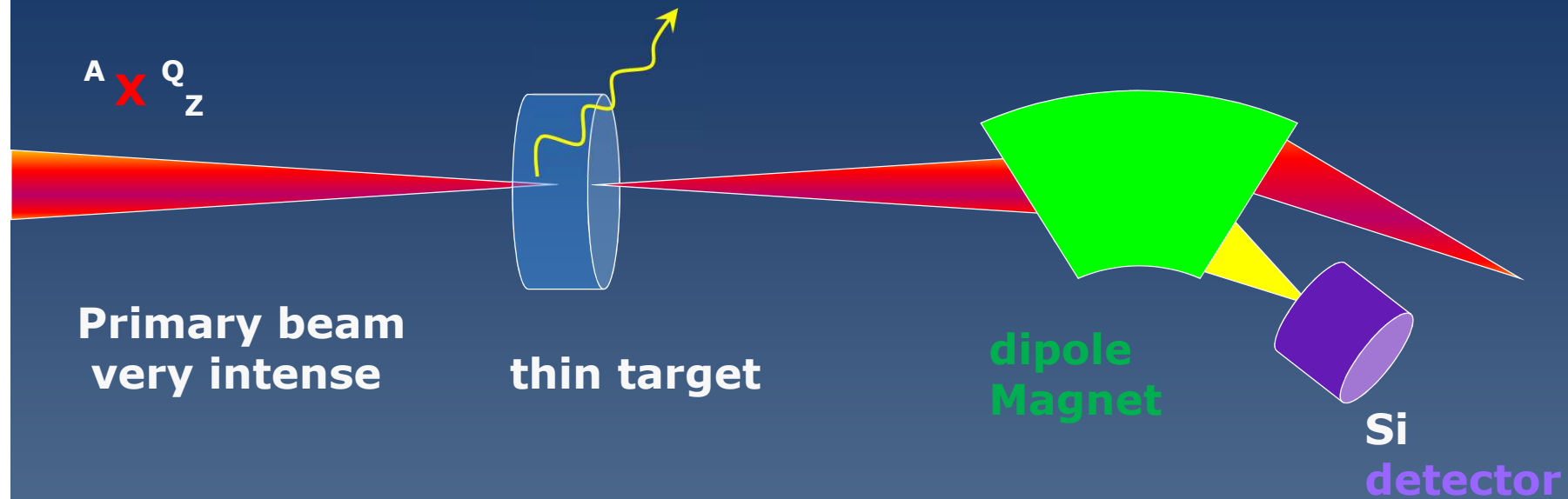


Reaction of interest, but



Reaction products not identified, ion energy not measured

An **other** experiment with a thin target & and a spectrometer

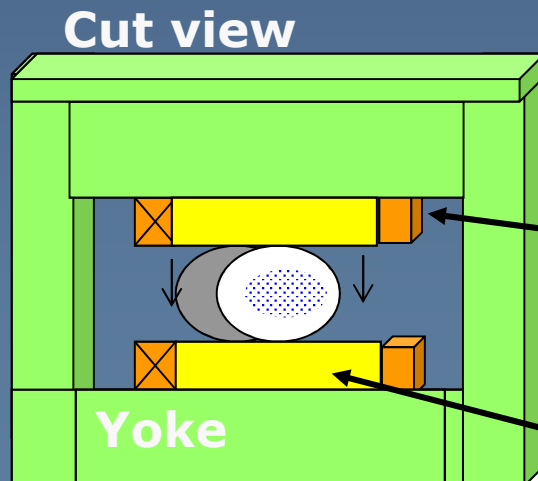
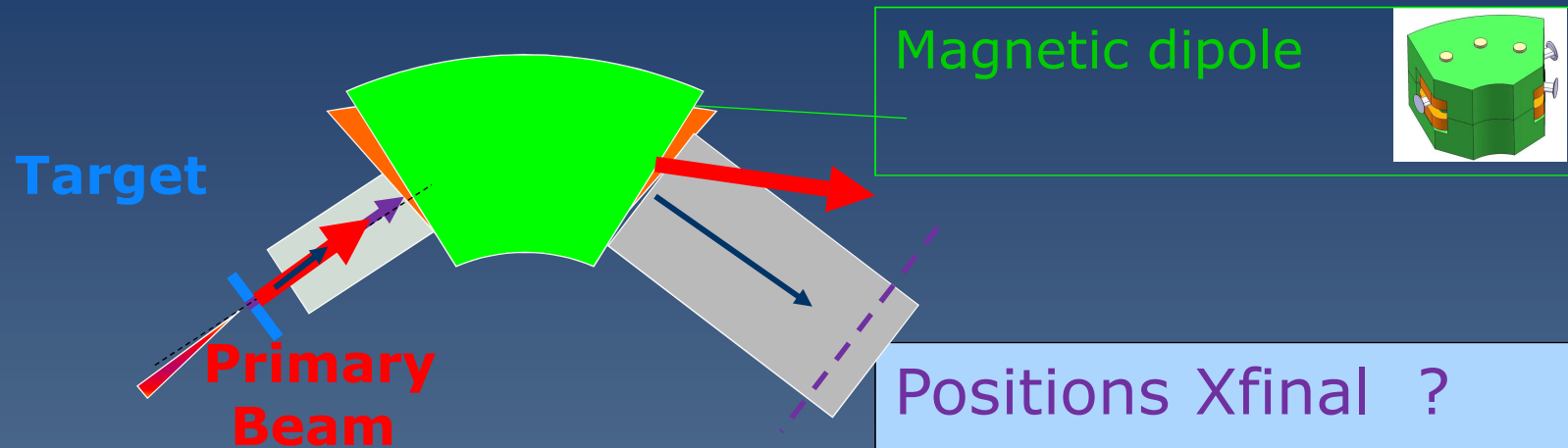


Electromagnetic spectrometer

- Eliminate **primary beam** ($\sim 10^{11-13}$ particles per second)
- Help to identify **the reaction products**
- Measure **Energy** with very good resolution
- Select **very rare events** (selectivity)

Let's design a simple **Magnetic** spectrometer

1) dispersion of the particles as function of M, v, \dots

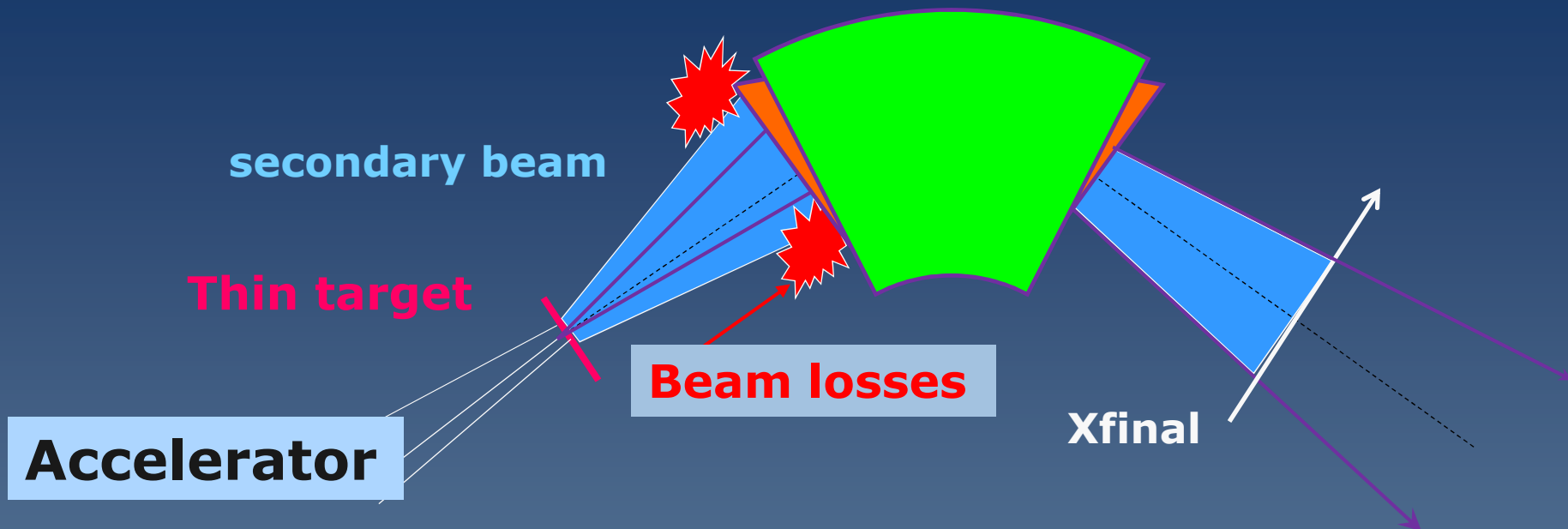


MAGNETIC DIPOLE : $B_y = \text{Constant}$
Coils (a current i induces and magnetic induction in the pole)

Yoke (guide the field lines to the pole)

2 flat poles : $B_y = \text{Constant}$

2 problems with 1 dipole magnet : Acceptance & identification



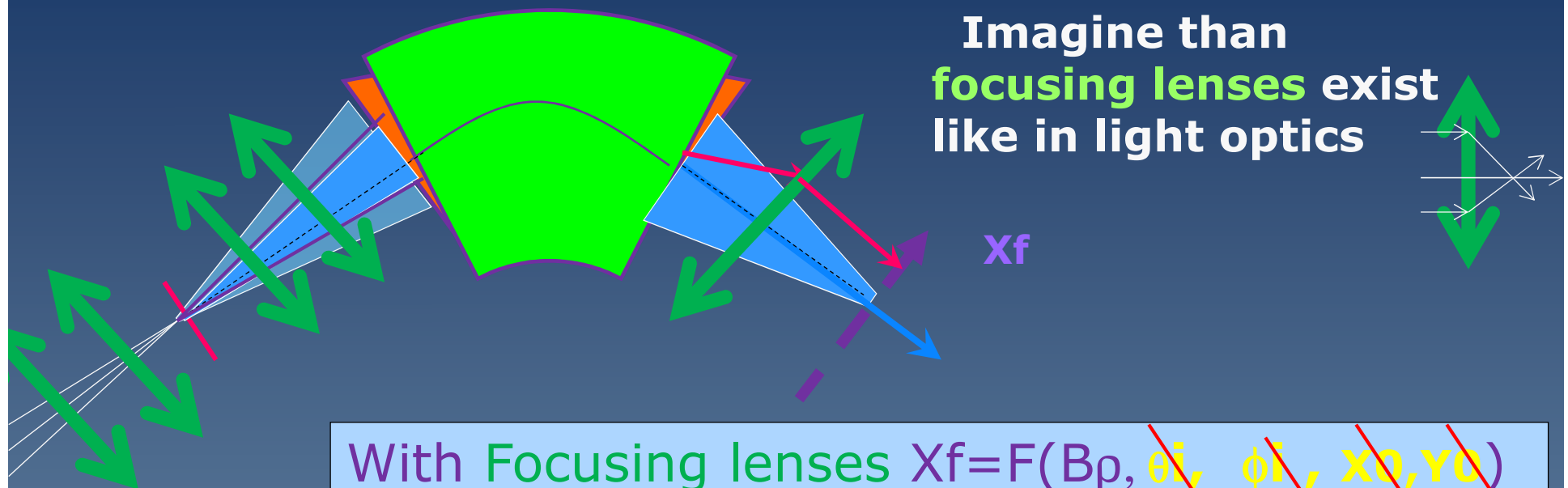
- 1: Many particles are lost in the magnet (very bad)
- 2: Trajectories are complex (bad)

$$X_{\text{final}} = f(B_{\rho}, \theta_i, \phi_i, X_0, Y_0)$$

- Final position Xf depend on the
 - B_{ρ} (good for identification or separation)
 - position & Angle after the reaction (bad)

Beam divergence after target

2 problems solved with **focusing lenses**



Imagine than **focusing lenses** exist like in light optics

With Focusing lenses $X_f = F(B_p, \theta_i, \phi_i, x_0, y_0)$

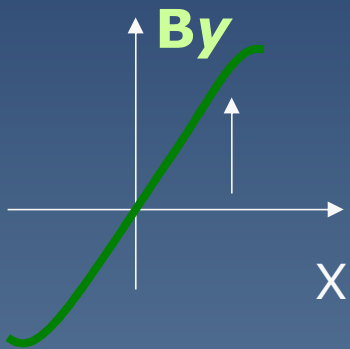
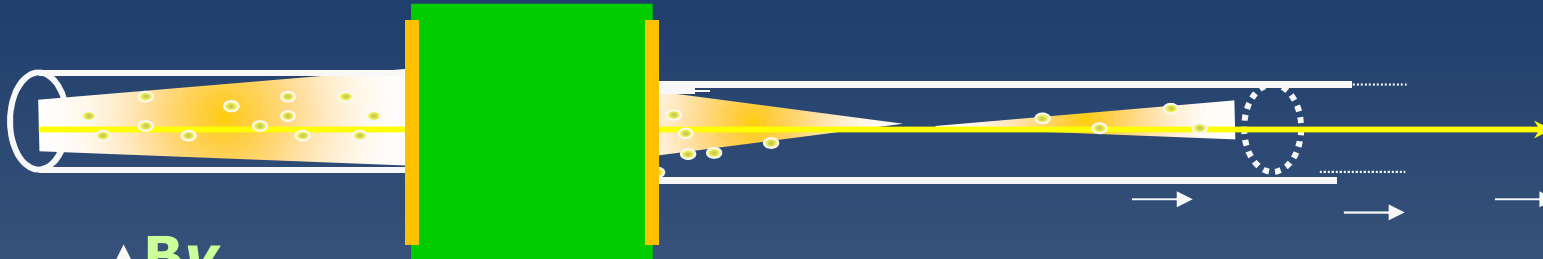
less unknowns ! **Less beam losses!!**

At one location s (the detector location, called **focal plan**)
 The trajectoires are independant of the angles θ_i, ϕ_i
 And the initial position is $x_0=0, y_0=0$

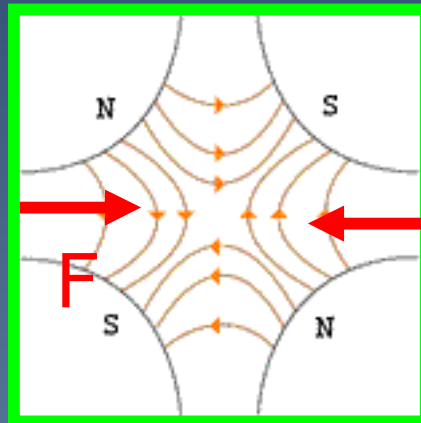
$$X_f = F(B_p, \theta_i, \phi_i, x_0, y_0)$$

How to construct a *Focusing lens* for ions :
Magnet with 4 poles (+,-,+,-)

$$F = q (v \times B)$$



$$G = dBy/dx$$

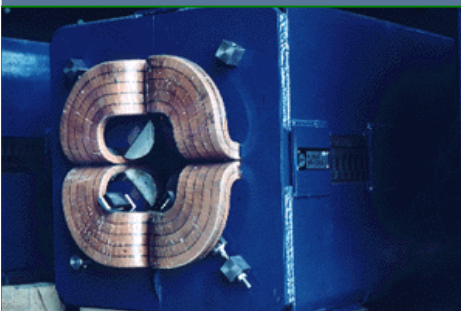


4 coils

+4 hyperbolic poles

$$By = G \cdot X$$

G is called GRADIENT
[Tesla/m]

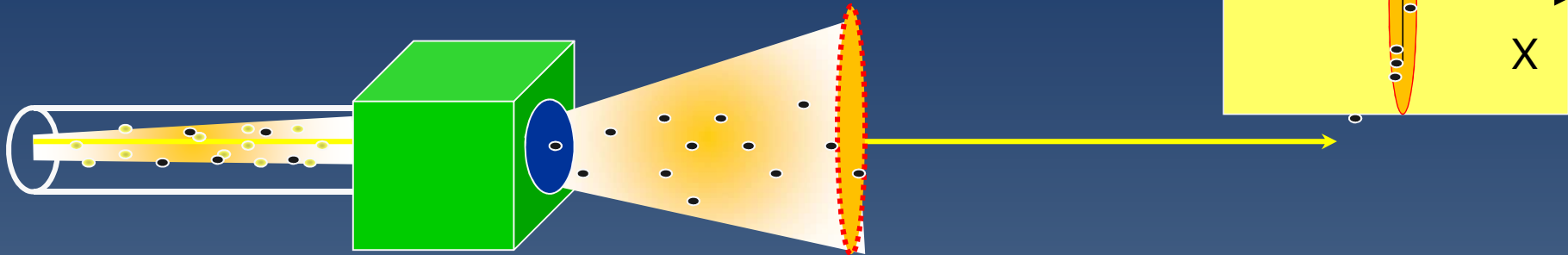


B.Jacquot // Ganil

The quadrupole magnet is **focusing**
in **HORIZONTAL PLAN**

Nota: In the center, the force is zero

*A quadrupole magnet
Focusing lens in horizontal
But defocusing in vertical*



The beam becomes narrow in X and large in Y

$$B_x = G \cdot Y$$

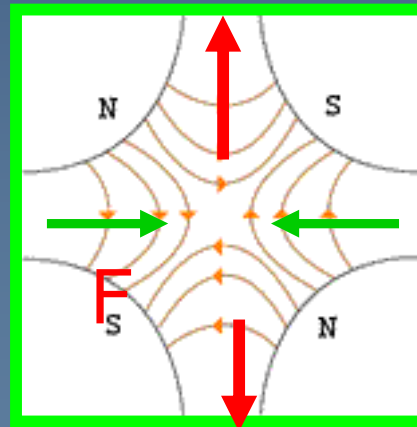
$$B_y = G \cdot X$$

$$B_s = 0$$

Focusing in X ($G > 0$)

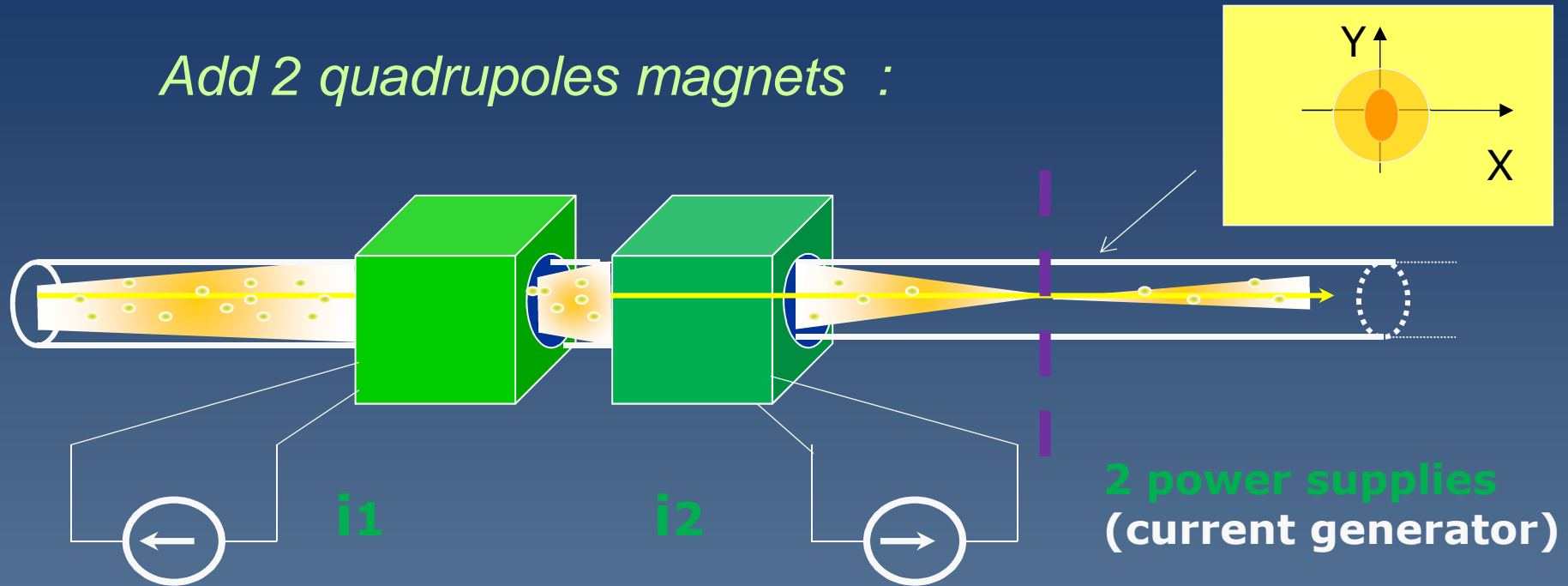
Defocusing in Y ($G > 0$)

Since the Force is defocusing in vertical



How to construct a Focusing lens System In horizontal AND vertical plan

Add 2 quadrupoles magnets :



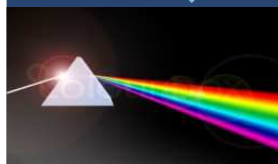
If you tune i_1 and i_2 with opposite polarities, the beam can be focused in X and Y

Beam optics (basics)



◆ Focalisation with quadrupoles

DONE



▼ Dispersion with dipole

DONE

Magnetic rigidity : $B\rho = \gamma Mv/Q = P/Q$

DONE

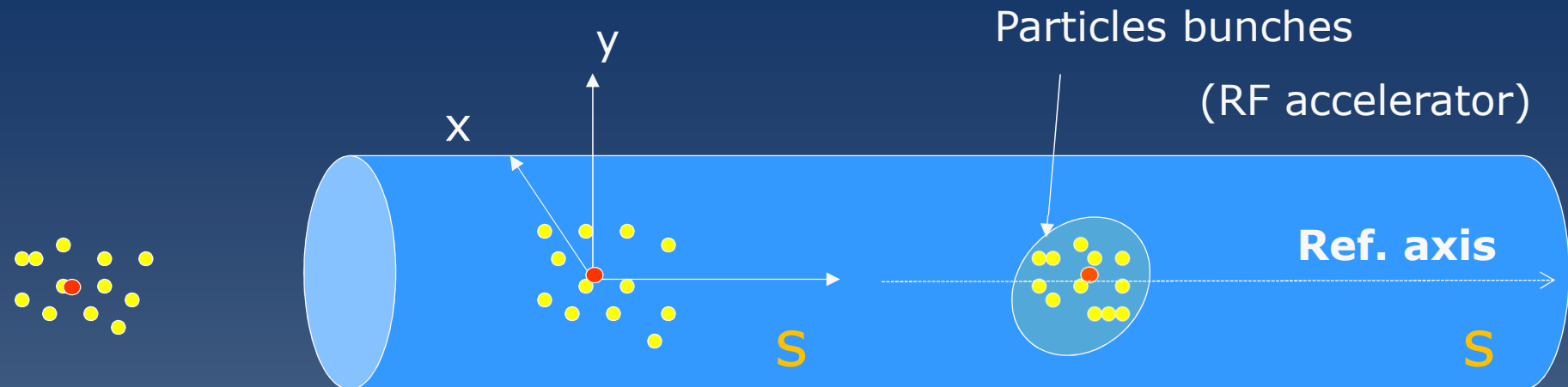
- Particles coordinates
- Equations in field B & E
- 1st order approximation : Optical Matrices



- Resolution
- Angular Acceptance
- $B\rho$ Acceptance



Beam Optics convention : Particle coordinates



Particle coordinates ? (energy, velocity, angle, $B\rho$, ??)

DEFINE A REFERENCE PARTICLE ($x_0, y_0, B\rho_0, t_0$)

At a given S , a particle is described with **6 coordinates** :

2 positions $(X-X_0), (Y-Y_0)$

+ 2 angles θ, ϕ

+ rigidity $B\rho = B\rho_0 (1+\delta)$

+ $(t - t_0)$ time advance

Optical convention :

Angle in Horizontal plan noted as

$$X' = dx/ds = \text{Tan } \theta$$

Angle in Vertical plan

$$Y' = dy/ds = \text{Tan } \phi$$

Time coordinate expressed in meter

$$L = v_0 (t - t_0)$$

Beam optics notation

The reference particle : $B\rho_0 = P_0/Q_0 = B_{dipole} \times R_{dipole}$

it is traveling in the **Center of the beam lines**

So $X_0=0$, $Y_0=0$

« angles » : $X'_0=0$, $Y'_0=0$

At the **location S_0** ,
a particle
 is represented
 by a vector $\vec{Z}(S_0)$

$\vec{Z} = (x, x', y, y', l, \delta)$
6Dim

$$\vec{Z} = \begin{pmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \\ z6 \end{pmatrix} = \begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ y \\ y' = \frac{dy}{ds} \\ l = v_0(T - T_0) \\ \delta = \frac{B\rho - B\rho_0}{B\rho_0} \end{pmatrix} = \begin{pmatrix} \text{horizontal displacement} \\ \text{horizontal "angle"} \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal difference} \\ \text{"momentum}(B\rho)\text{" deviation} \end{pmatrix}$$

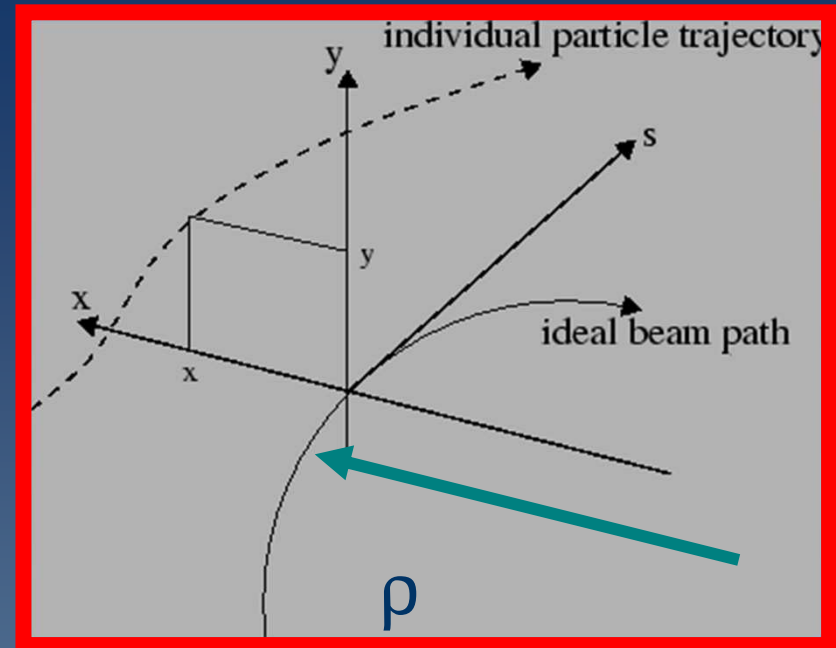
Trajectory equations for 1 particle

How to compute $x(s), y(s)$?

We use a

curvilinear Reference Frame

which follows the reference particle



$$\frac{d}{dt} [m\gamma \mathbf{v}] = q \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{d}{dt} = \dot{s} \frac{d}{ds}$$

Coordinate change $t \rightarrow s$

$x(t), y(t) \Rightarrow x(s), y(s)$

We want to compute x, y
at a detector location $s=s_0$

$$\frac{d}{ds} [m\gamma \mathbf{v}] = \dots$$

Trajectories : exact equations

$$\frac{d}{ds} \left[m\gamma \dot{x} \right] = m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) + q(t' E_x + y' B_s - \dot{s} \left(1 + \frac{x}{\rho} \right) \cdot B_y)$$

$$\frac{d}{ds} \left[m\gamma \dot{y} \right] = q(t' E_y + \left(1 + \frac{x}{\rho} \right) \cdot B_x - x' \cdot B_s)$$

$$\frac{d}{ds} \left[m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) \right] = -\frac{m\gamma \dot{x}}{\rho} + q(t' E_s + x' \cdot B_y - y' \cdot B_x)$$



Trajectory simulation ($x(s)$, $y(s)$)

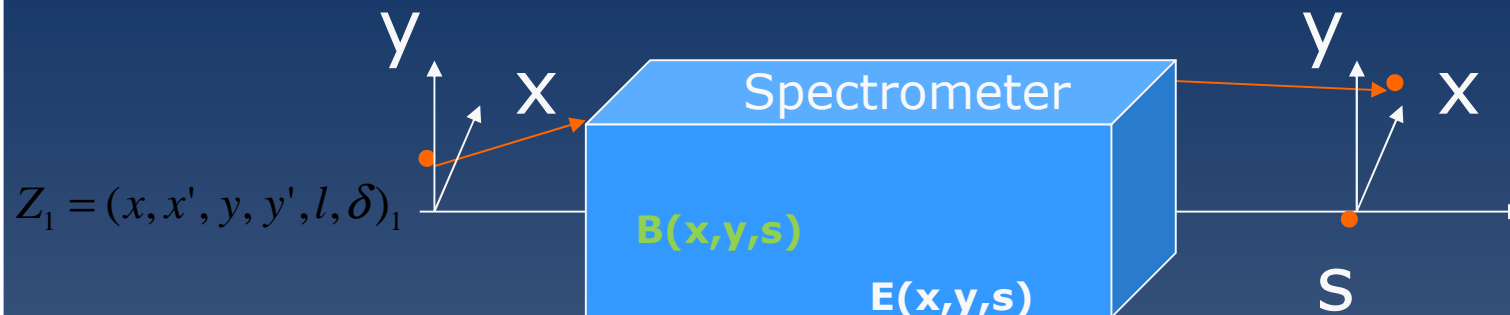
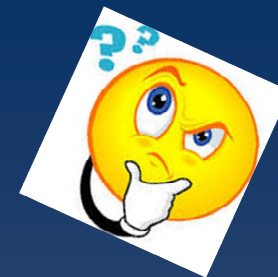
- 1) knowing $B(x,y,s)$ AND $E(x,y,s,t)$ [field map 3D]
- 2) Integrate the equations for **ALL the particles**
(computer+ Numerical method: Runge-kutta)

Generally we can do simpler

Matrix approach (1st order approximation)



Beam optics with Matrices



$$\begin{aligned} \mathbf{Z}_2 &= f_{1 \rightarrow 2} (\mathbf{Z}_1, B, E, l, \dots) \\ &= R_{1 \rightarrow 2} \cdot \mathbf{Z}_1 + O(\mathbf{Z}_1^2) + \dots \\ &\approx R_{1 \rightarrow 2} \cdot \mathbf{Z}_1 \end{aligned}$$

Exact Dynamic (non linear)

Taylor expansion

(X and Y Are small..)

Linear dynamics

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_2 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1$$

$$\begin{aligned} l &= v_0(t - t_0) \\ \delta &= \frac{B\rho - B\rho_0}{B\rho_0} \end{aligned}$$

The simplest transport Matrice: Rmatrix for a straight section L (drift)

Particle Evolution in **drift** length between s_1 & s_2 :

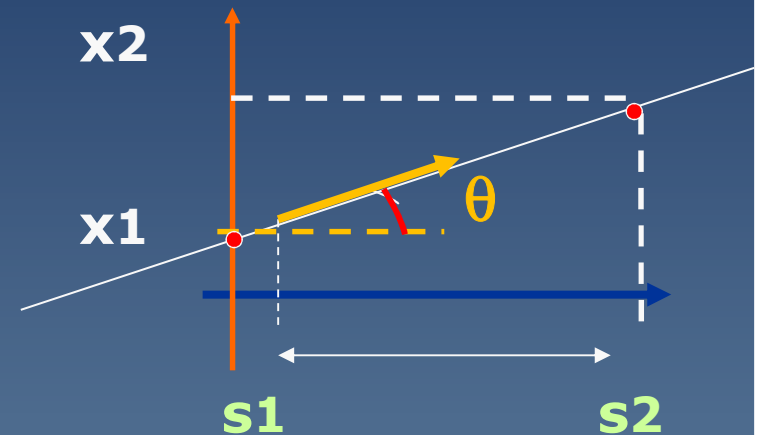
$$\mathbf{x} = \mathbf{x}(s) \quad \text{??????}$$

$$x_2 = x_1 + \tan(\theta_1)(s_1 - s_2)$$

$$\theta_1 = \theta_2$$

nota: $\tan(\theta_1) = \Delta x_1 / \Delta s = x_1'$

and $(s_2 - s_1) = L$



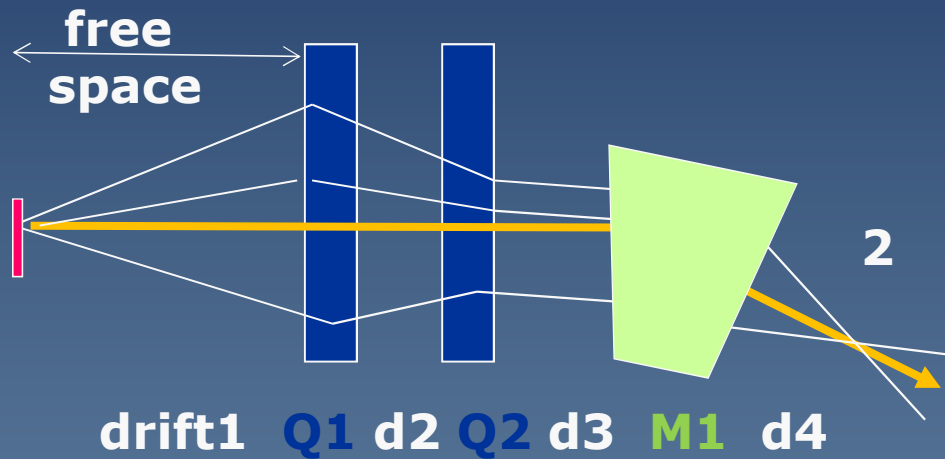
$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \\ \dots \end{pmatrix} = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

$$R_{d1} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

More on Transport Matrices:

how to compute the Rmatrix for a spectrometer ?

The **total transport** matrix **R** is the **product** of the matrices representing each elements (drift ,quad, dipole)



Quad matrix

$$R_{M1} = \begin{bmatrix} \cos k_x L & \frac{\sin k_x L}{k_x} & 0 & 0 & 0 & M_{16} \\ -k_x \sin k_x L & \cos k_x L & 0 & 0 & 0 & M_{26} \\ 0 & 0 & \cos k_y L & \frac{\sin k_y L}{k_y} & 0 & 0 \\ 0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\ M_{26} & M_{16} & 0 & 0 & 1 & M_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$k_x = GL / B\rho_0$

$$R_{d1} = \begin{bmatrix} 1 & L1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L1/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Free space:
Drift
Matrix

Matrix product $R = d4 \cdot R_{M1} \cdot R_{d3} \cdot R_{Q2} \cdot R_{d2} \cdot R_{Q1} \cdot R_{drift1}$

The transport Matrix R allows the computation of the coordinates of a particle at the end of a spectrometer

$$\begin{array}{c} \longrightarrow \\ Z_{in} = (x, x', y, y', l, \delta)_0 \end{array} \quad \text{at the entrance}$$

$$\begin{array}{c} \longrightarrow \\ Z_{out} = (x, x', y, y', l, \delta)_1 \end{array} \quad \text{at the exit}$$

$$\begin{array}{c} \longrightarrow \quad \longrightarrow \\ Z_{out} = R \cdot Z_{in} \end{array}$$

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & R_{55} & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = v_0(t - t_0)$$

$$\delta = \frac{p - p_0}{p_0}$$

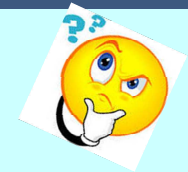
Interpretation of R

$$R_{ij} = \left(\frac{\partial Z_i \text{ out}}{\partial Z_j \text{ in}} \right)$$

ex:

$$R_{11} = \left(\frac{\partial Z_1}{\partial Z_1} \right) = \left(\frac{\partial x \text{ out}}{\partial x \text{ in}} \right) \quad R_{12} = \left(\frac{\partial Z_1}{\partial Z_2} \right) = \left(\frac{\partial x \text{ out}}{\partial x' \text{ in}} \right)$$

$$R_{16} = \left(\frac{\partial Z_1}{\partial Z_6} \right) = \left(\frac{\partial x \text{ out}}{\partial \delta \text{ in}} \right)$$



The transport Matrix $R = R_{ij}$ is related to

- spectrometer geometry
- **tuning** of the quadrupoles

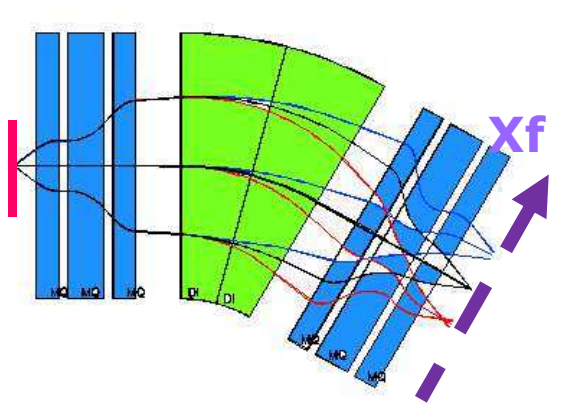
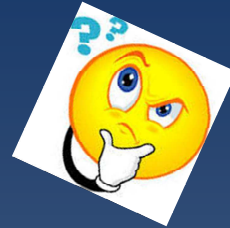
SPECTROMETER TRANSPORT MATRIX R

allow the simulation of 1 trajectory (easily)

$$\frac{d}{ds} \left[m\gamma \dot{x} \right] = m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) + q(t' E_x + y' B_s - \dot{s} \left(1 + \frac{x}{\rho} \right) \cdot B_y)$$

$$\frac{d}{ds} \left[m\gamma \dot{y} \right] = q(t' E_y + \left(1 + \frac{x}{\rho} \right) \cdot B_x - x' B_s)$$

$$\frac{d}{ds} \left[m\gamma \dot{s} \left(1 + \frac{x}{\rho} \right) \right] = -\frac{m\gamma \dot{x}}{\rho} + q(t' E_s + x' B_y - y' B_x)$$



$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{FINAL} = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_{TARGET}$$



Typical spectrometer Matrix is simple



$$\mathbf{X}_{Final} = R_{11} \mathbf{X}_{target} + R_{16} \delta$$

$$\approx R_{16} \delta$$

$$R_{16} = \left(\frac{\partial x_F}{\partial \delta_{Target}} \right)$$

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

$$B\rho_0 = \mathbf{B}_{dipole} \cdot R_{dipole}$$

The **beam size** : important for the design

- A **particle** has 1 trajectory : $\vec{Z} = \vec{z}(s)$

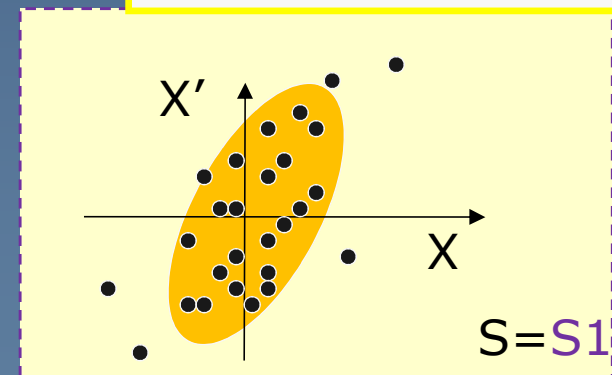
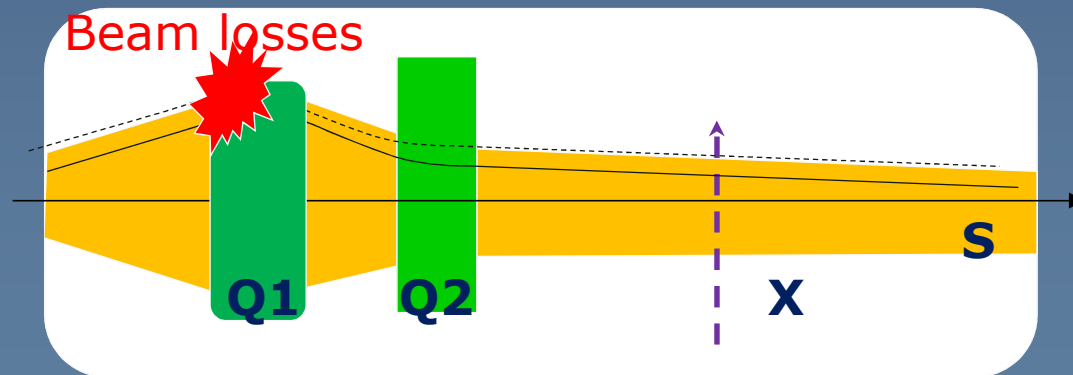
We are not interested by only 1 trajectory/1 particle

*A beam is an ellipsoid in 6D with a **given size***

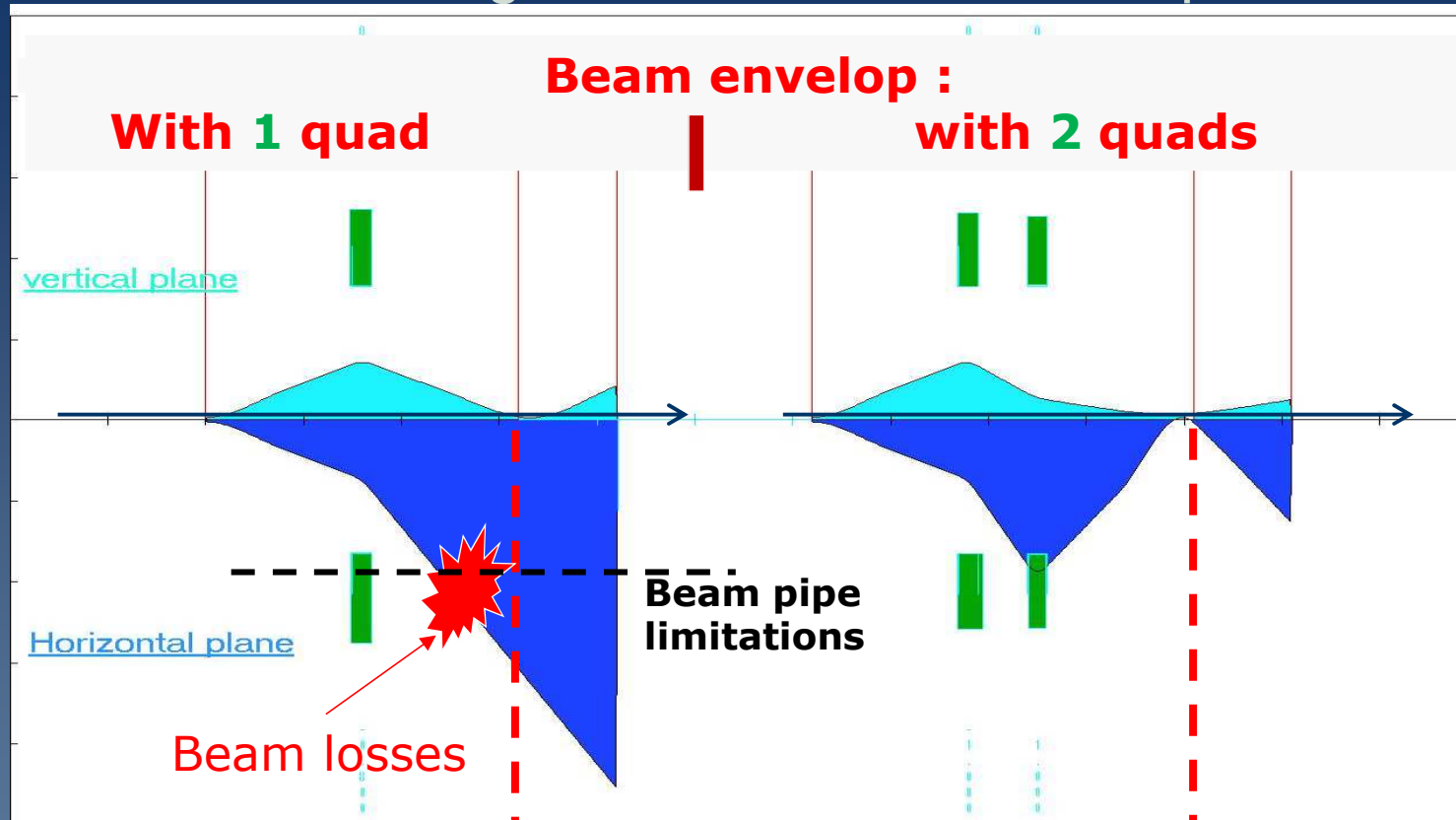
*The **beam size(width)** has to be simulated to avoid **beam losses***

σ_x (horizontal width) , σ_y (vertical width)

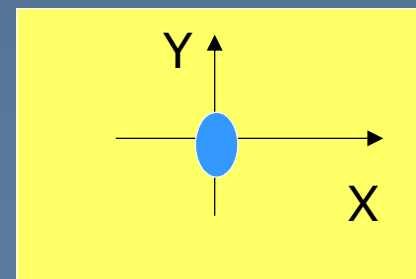
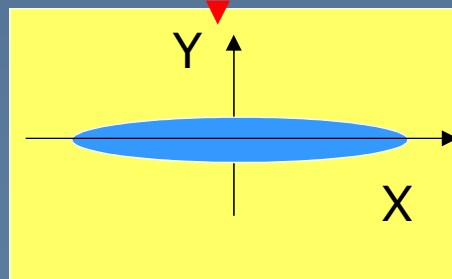
$$\sigma_x^2 = \frac{1}{N} \sum_{\alpha=1, \dots, N} x_\alpha^2$$



Focusing a beam in a simulation get a small size at some point S

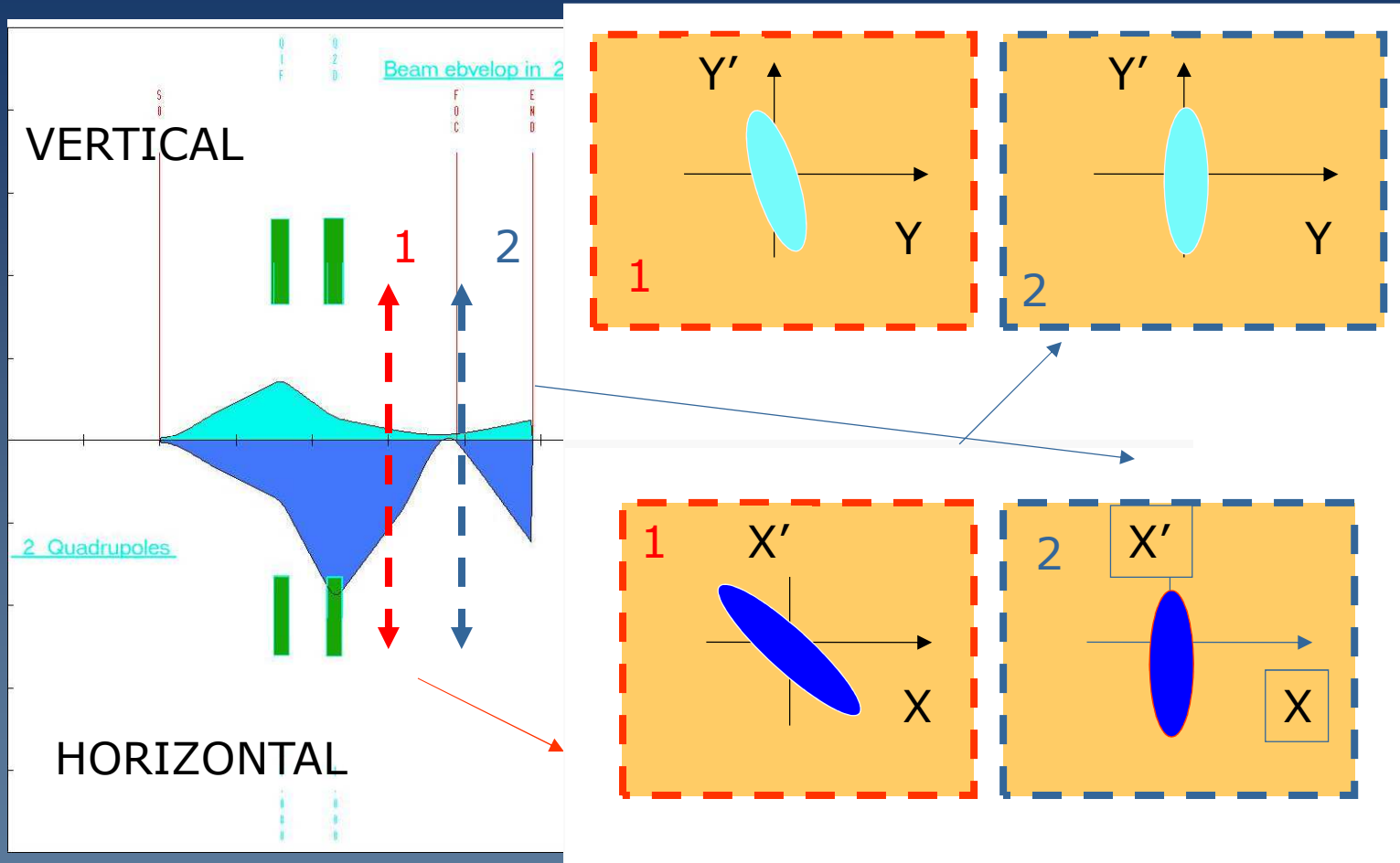


Adjust
Quad gradient
 $G_q = dB_y/dx$



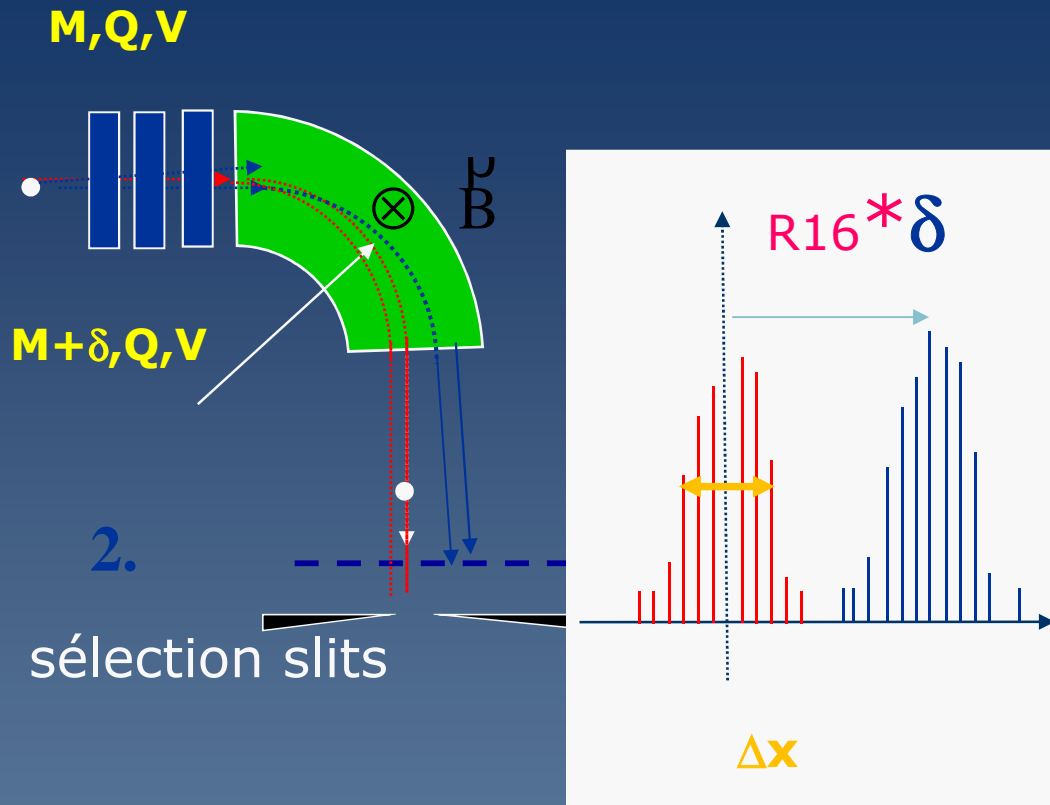
Focusing in X and Y : at less 2 quads required

Angular distribution (x') in a beam line ?
 The beam ellipse is rotating in $(x, x'=dx/ds)$



...The **Area** of the beam ellipse (x, x') is a constant in a beam line... **but, Area is not constant in a target**

Resolution of a magnetic spectrometer



particles are separated

IF $R_{16} * \delta > 4 \sigma_x$

Resolution = $4 \sigma_x / R_{16}$
 = Minimal difference in $B\rho$
 for the identification
 or for separation

$$R_{16} = \left(\frac{\partial Z_1}{\partial Z_6} \right) = \left(\frac{\partial x_{out}}{\partial \delta_{in}} \right)$$

R=1/100 Resolution means :
 capacity for a spectrometer to
 distinguish two beams with
 1% $B\rho$ difference

The resolution R (separation)

is **optimal** at the **focus point** (size is minimal)

The 2 beams with \neq rigidities

$$B\rho_{\text{ref}} = B\rho_0 = B \times R_{\text{dipole}}$$

$$B\rho = B\rho_0(1-\delta)$$

The 2 beams are separated

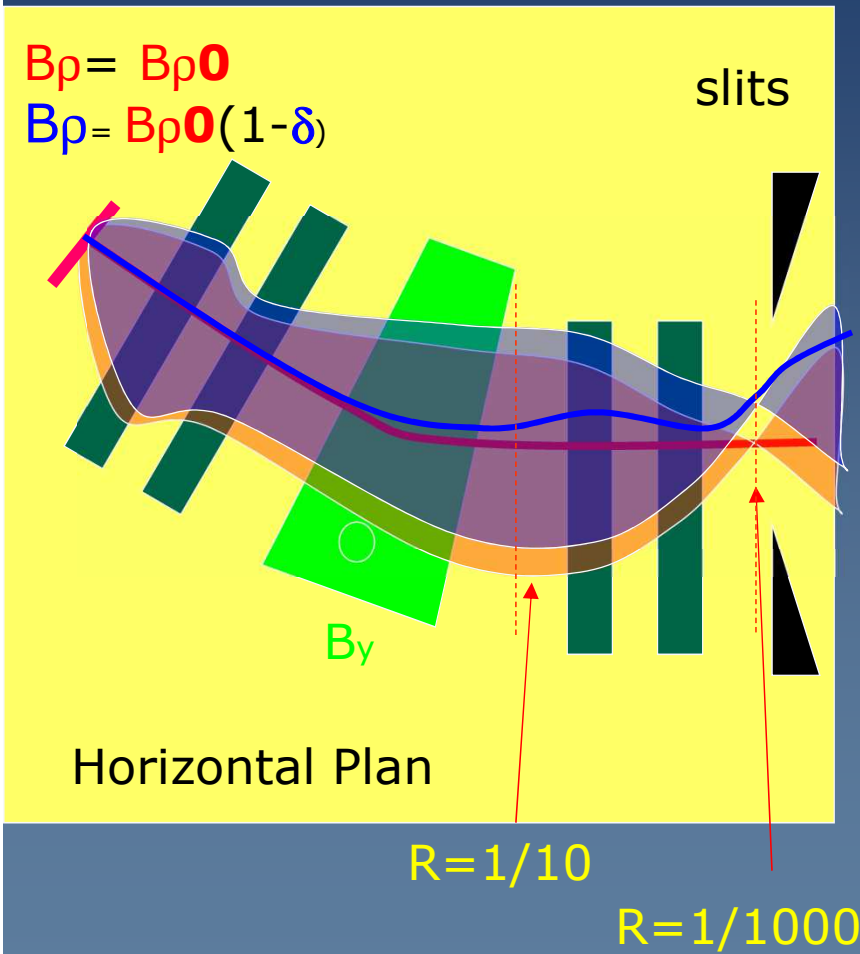
« at the focal plan »

But not everywhere !!

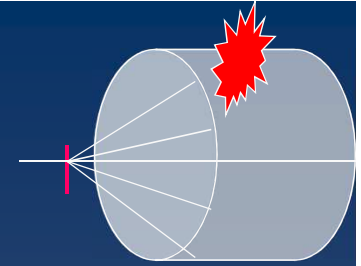
Resolution ($R = \sigma_x / R_{16}$) is **optimal**

When σ_x is **small**

and R_{16} (dispersion) is **large**

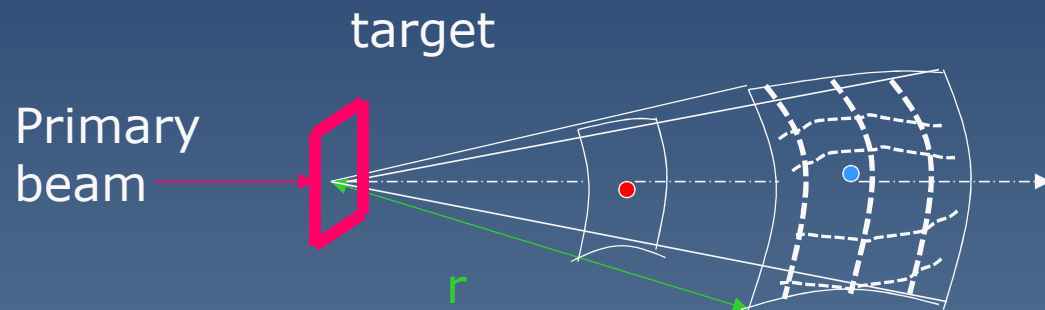


Angular acceptance



The **reaction products** exit from the target with an
Angular dispersion

Vacuum chamber limitation induces **beam losses** = less transmission



The acceptance
is measured in steradian.

$$d\Omega(\text{strd}) = \frac{dS}{r^2}$$

dS

Example: If particles inside $\pm 50\text{mrd}$
(Horizontal & vertical) are transmitted

Acceptance is $d\Omega \approx 0.01\text{strd} = 10 \text{ mstrd}$

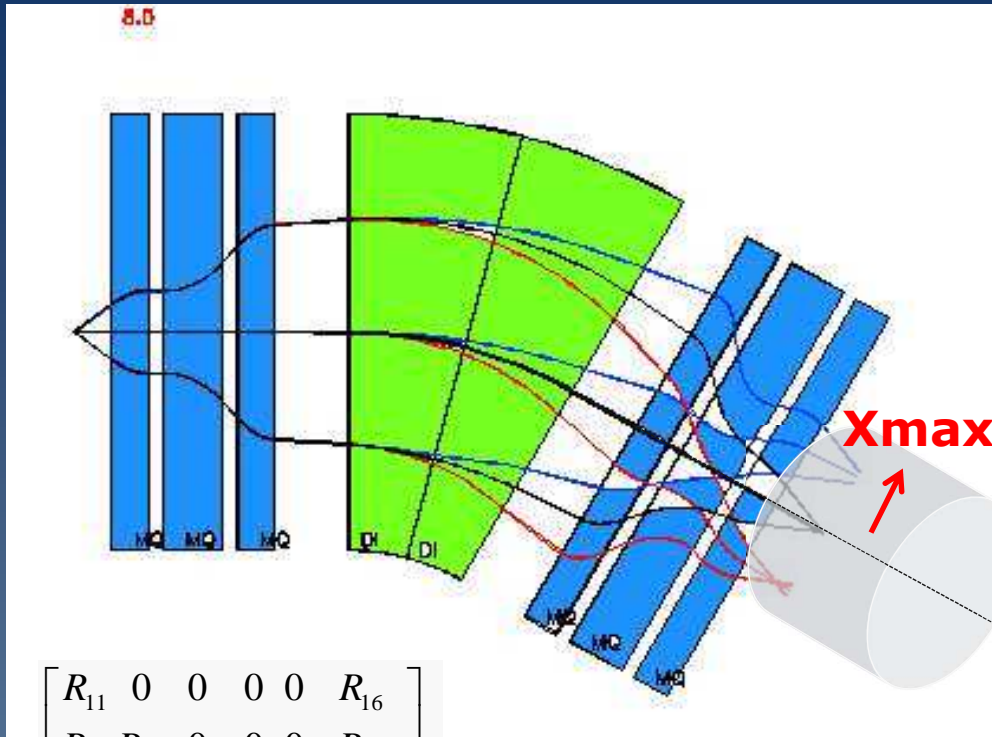
at $r=1\text{m}$

$$dS \# 0.1\text{m} * 0.1\text{m} = 0.01 \text{ m}^2$$

Beam losses



« B_p » Acceptance



The particles are dispersed by dipole magnets with

$$\delta = [B_p - B_{p0}] / B_{p0}$$

$$X_{\text{final}} = R_{16} \delta$$

Beam pipe limit: Xmax

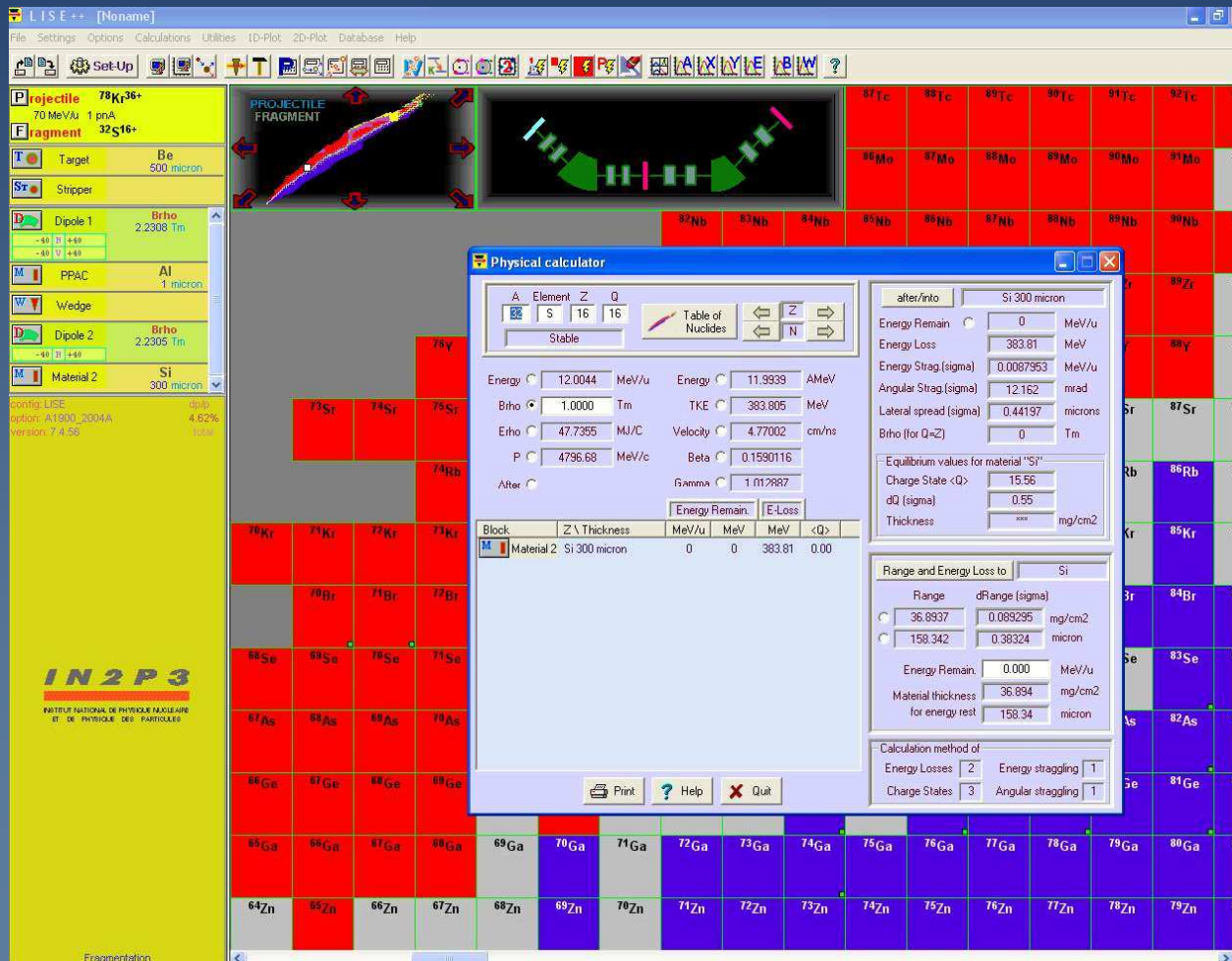
$$\begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_p \text{ Acceptance} = \pm X_{\text{max}} / R_{16}$$

Example : If $R_{16}=5 \text{ cm}/\%$ and $X_{\text{max}}=10 \text{ cm}$

$$B_p \text{ Acceptance} = \pm 2 \%$$

How to simulate an experiment with a spectrometer in nuclear physics



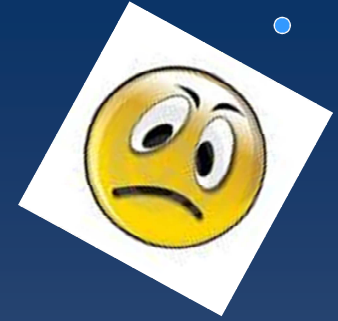
LISE++

code*

Tarasov et Al.

**To be
Downloaded**

HOMEWORK :



Exercise 1: Imagine a spectrometer with a dispersion $R_{16}=2 \text{ m } (=2\text{cm}/\%)$ and beam width $\sigma_x = 0.5 \text{ mm}$ on the focal plan detector,
What is the resolution R in B_ρ ?

Exercise 2 :

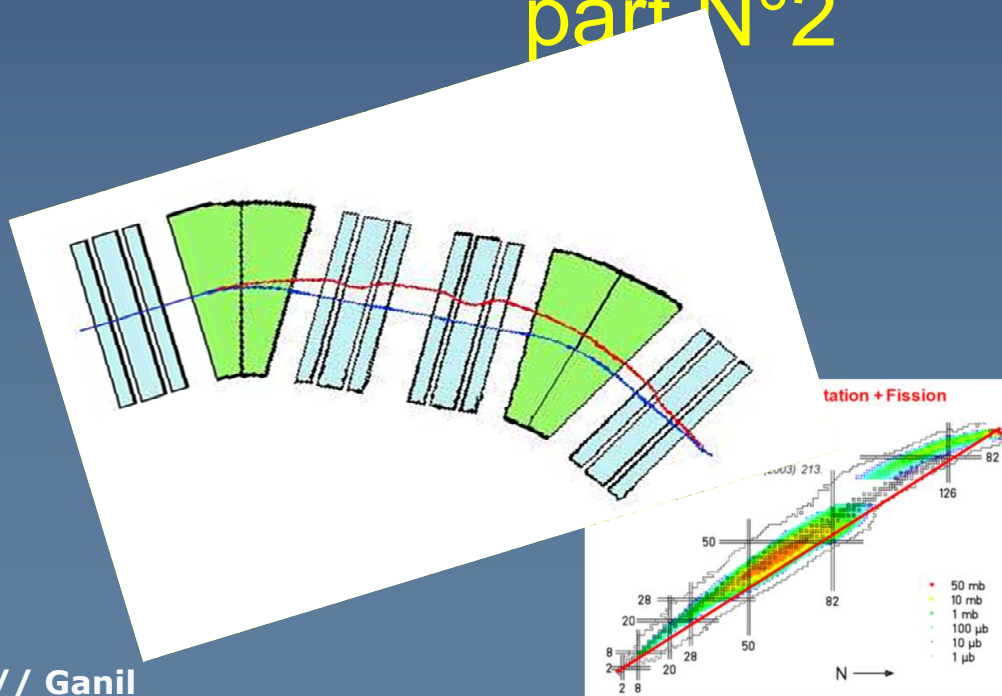
A spectrometer ($R_{16}=1.5 \text{ cm}/\%$) is tuned for $B_{\rho 0}=2.0 \text{ T.m}$
A particle arrives on the focal plane at $X_f=3\text{cm}$,
What is the particle rigidity?

Exercise 3 :

How to measure the dispersion (R_{16}) in a spectrometer ?

II) Spectrometers with accelerator beams

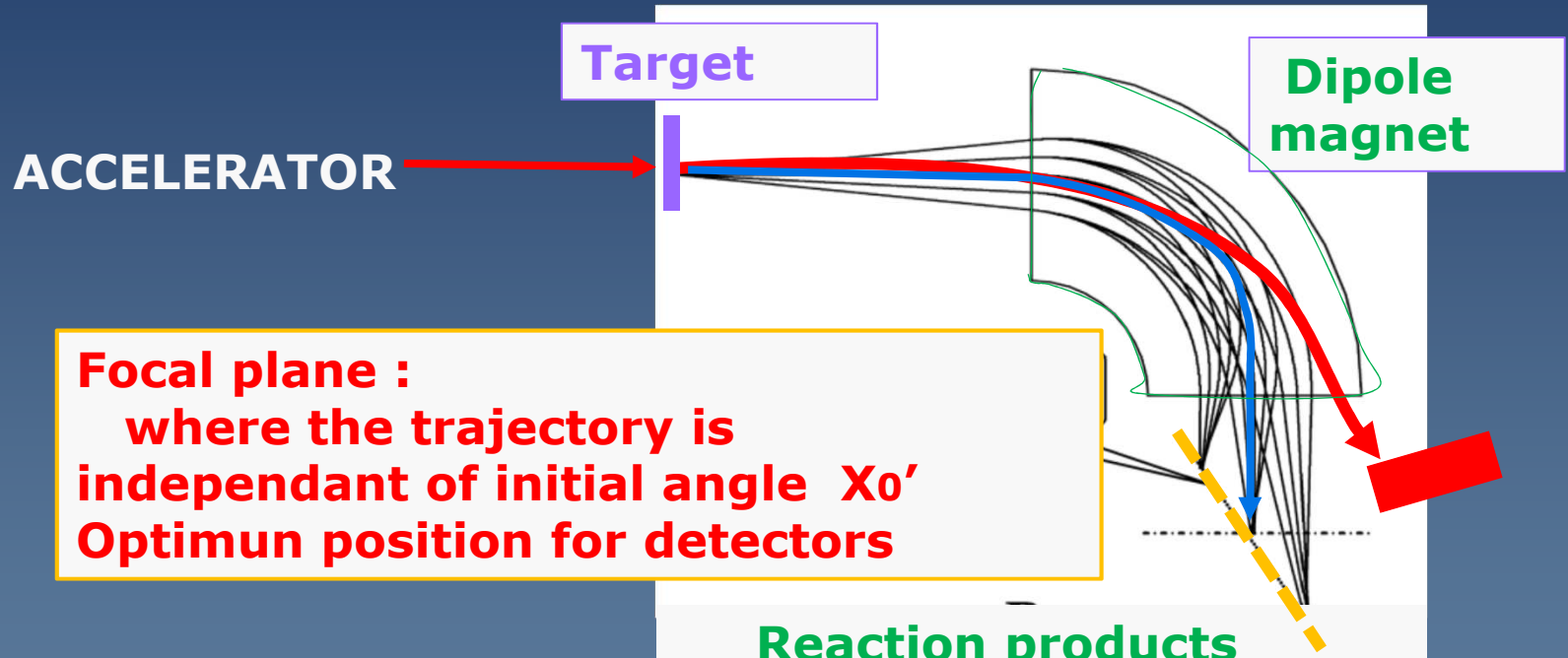
part N°2



Magnetic Spectrometer recap

- The need of focalisation (quadrupole)
- Magnetic rigidity define the trajectory
- Dynamics can be approximated with a matrix R

$$B\rho \stackrel{def}{=} \gamma \frac{mv}{q}$$



Focal plane :
 where the trajectory is
 independant of initial angle X_0'
 Optimun position for detectors

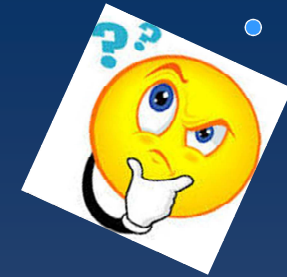
Reaction products

$$X_f \sim F(B\rho)$$

$$X_f = R_{11} X_{target} + R_{16} \delta$$

$$\approx R_{16} (B\rho - B_0 R_0) / B_0 R_0$$

Beam optics coordinates



- At the location S , a particle is represented by a vector $\mathbf{Z}(s) = (x, x', y, y', l, \delta)$

$$\begin{matrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \\ z6 \end{matrix} \stackrel{\rho}{Z} = \begin{pmatrix} x \\ x' = \frac{dx}{ds} \\ y \\ y' = \frac{dy}{ds} \\ l = v_0(T - T_0) \\ \delta = \frac{B\rho - B\rho_0}{B\rho_0} \end{pmatrix} = \begin{pmatrix} \text{horizontal displacement} \\ \text{horizontal "angle"} \\ \text{vertical displacement} \\ \text{vertical angle} \\ \text{longitudinal difference} \\ \text{"momentun}(B\rho)" \text{ deviation} \end{pmatrix}$$

HORIZONTAL ANGLE

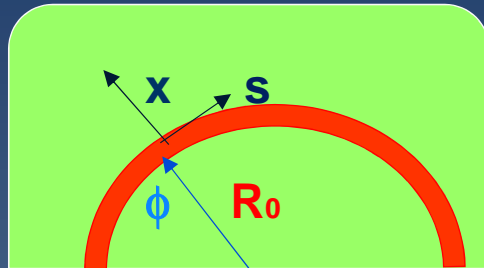
$$X' = dX/ds = \tan(\theta) \approx \theta$$

Rmatrix for a magnet ($\phi=180^\circ$ exemple)

Frame = attached to the **reference particle** (circle with $R=R_0$)

S = curvilinear absciss

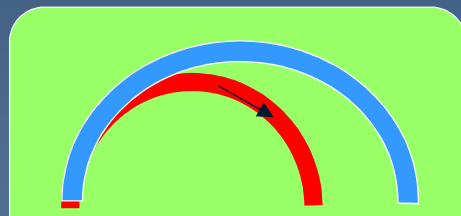
What is the position of a particle with ($x_0 \neq 0, x_0' \neq 0, B_\rho \neq B_{\rho 0}$)



*

$$x_{ref}(\theta = s/R) = 0$$

Reference ($x_0=0, x_0'=0, B_\rho=B_{\rho 0}=B_0 R_0$)



$R \neq R_0$

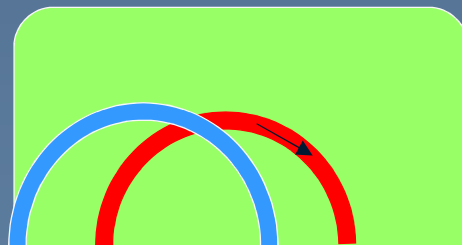
* **R16** : x final pos. if a different rigidity

$$X(\phi=180^\circ) - x_{ref} = 2 (R - R_0)$$

$$X(\phi) = (1 - \cos(\phi)) (R - R_0)$$

$$= (1 - \cos(\phi)) (B_\rho - B_{\rho 0}) / B_0 =$$

$$= R_0 (1 - \cos(\theta)) (B_{\rho 0} - B_{\rho 0}) / B_{\rho 0}$$



$x_0 \neq 0$

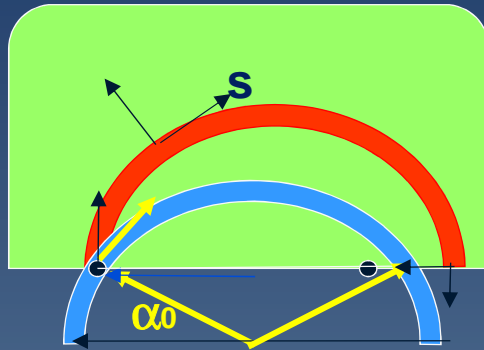
R11 : x final pos. if a different initial position

$$X(\phi=180^\circ) = -X(\phi=180^\circ) = -x_0$$

$$X(\phi) = \cos(\phi) x_0 = \mathbf{R11} x_0$$

Rmatrix for a magnet ($\phi=180^\circ$ exemple)

What is the position of a particle starting with an angle
($X'_0 = \tan(\phi_0)$)



R12= Trajectory with initial different angle
Arrive at different position

$$X(\phi=180^\circ) = 2R_0 (1-\cos(\alpha_0)) \# R_0 X'_0$$

$$X'_0 = \tan(\alpha_0)$$

$$X(\phi) \# R_0 \sin(\phi) X'_0$$

$$\begin{aligned} X_{\text{final}} &= -R_{11} x_0 + R_{12} X'_0 + R_{16} (B_{\rho 0} - B_{\rho 0}) / B_{\rho 0} \\ &= -\cos(\phi) x_0 + R_0 \sin(\phi) X'_0 + R_0 (1 - \cos(\phi)) \delta \end{aligned}$$

$$M_{\text{dipole}} = \begin{pmatrix} \boxed{\cos \phi} & \boxed{R \sin \phi} & 0 & 0 & \boxed{0} & \boxed{R(1 - \cos \phi)} \\ \boxed{-1/R \cdot \sin \phi} & \boxed{\cos \phi} & 0 & 0 & \boxed{0} & \boxed{\sin \phi} \\ 0 & 0 & \boxed{1} & \boxed{R\phi} & 0 & 0 \\ 0 & 0 & \boxed{0} & \boxed{1} & 0 & 0 \\ \boxed{-\sin \phi} & \boxed{-R(1 - \cos \phi)} & 0 & 0 & \boxed{1} & \boxed{R\phi/\gamma^2 - R(\phi - \sin \phi)} \\ \boxed{0} & \boxed{0} & 0 & 0 & \boxed{0} & \boxed{1} \end{pmatrix}$$

Transport matrix
For a dipole with
angle ϕ

The **R** matrix of spectrometer

: first order theory

A spectrometer generally

- A) starts with a focus (on target)
- B) End up with a focus ($R_{12}=R_{34}=0$)
- C) The spectrometer is chromatic ($R_{16} \neq 0$)

typical matrix (8 coefficients)

$$\begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_1 = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ - & - & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{bmatrix}_0$$

$$l = v_0(t - t_0) \\
 \delta = \frac{B\rho - B\rho_0}{B\rho_0}$$

R_{16} is called dispersion

R_{11} is called MAGNIFICATION



$$R_{11} = \Delta X_F / \Delta X_{\text{Target}}$$

Coordinates
At focal (detectors)

Coordinates
on target

$$x^F \approx \sum_{j=1 \dots 6} R_{1j} Z_j^0 = R_{11} \cdot x_0 + R_{12} \cdot x'_0 + R_{13} \cdot y_0 + R_{14} \cdot y'_0 + R_{15} \cdot l^0 + R_{16} \cdot \delta^0$$

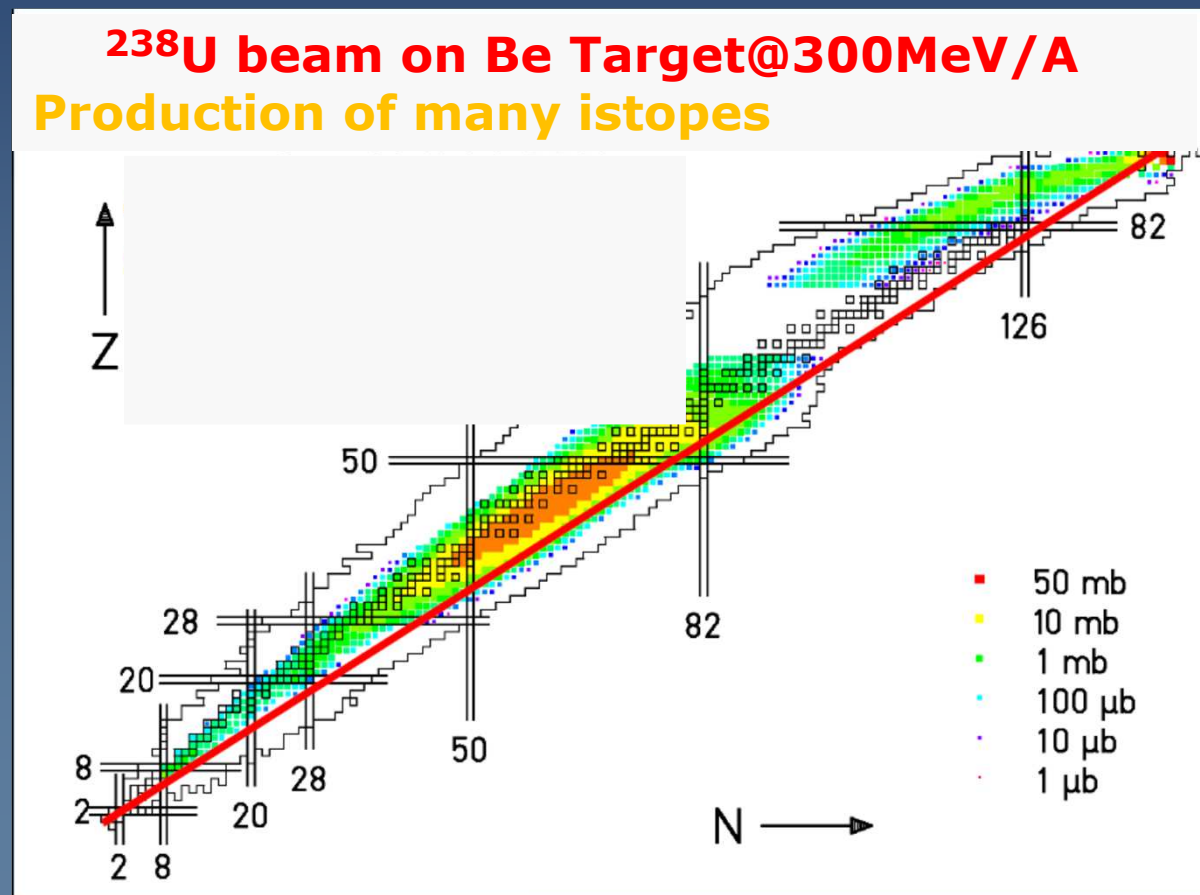
Fragment Separators : selection and identification of radioactive isotopes ($T_{1/2} \sim 10\text{ms}-1\text{s}$)

Reaction : fragmentation or U fission

accelerator ion beams at **100-1000 MeV/A**

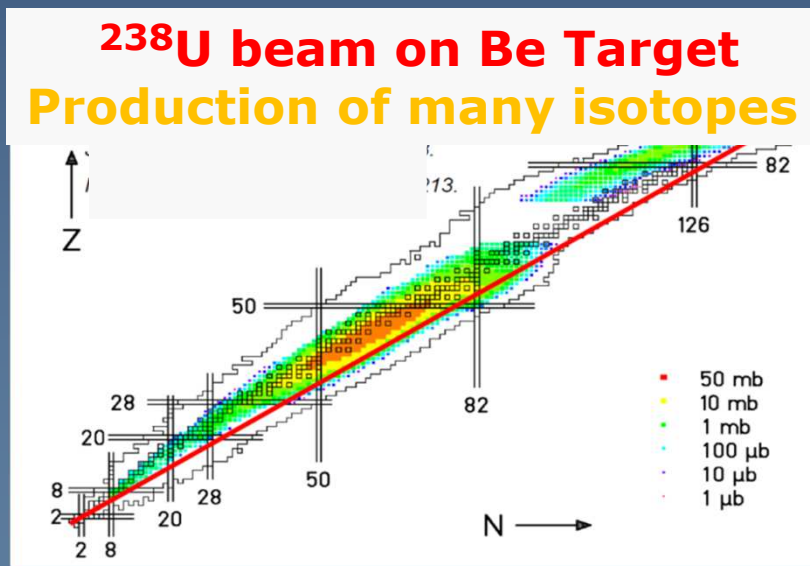
$\beta=0,40 -0,85$

GOAL
The production & study of exotic nuclei beyond the region of nuclei known today



Fragment Separators : selection of a specific isotopes

- 1) Primary beam suppression (Separator)
- 2) Identification of particles
- 3) **purification** (selection of some reaction products)

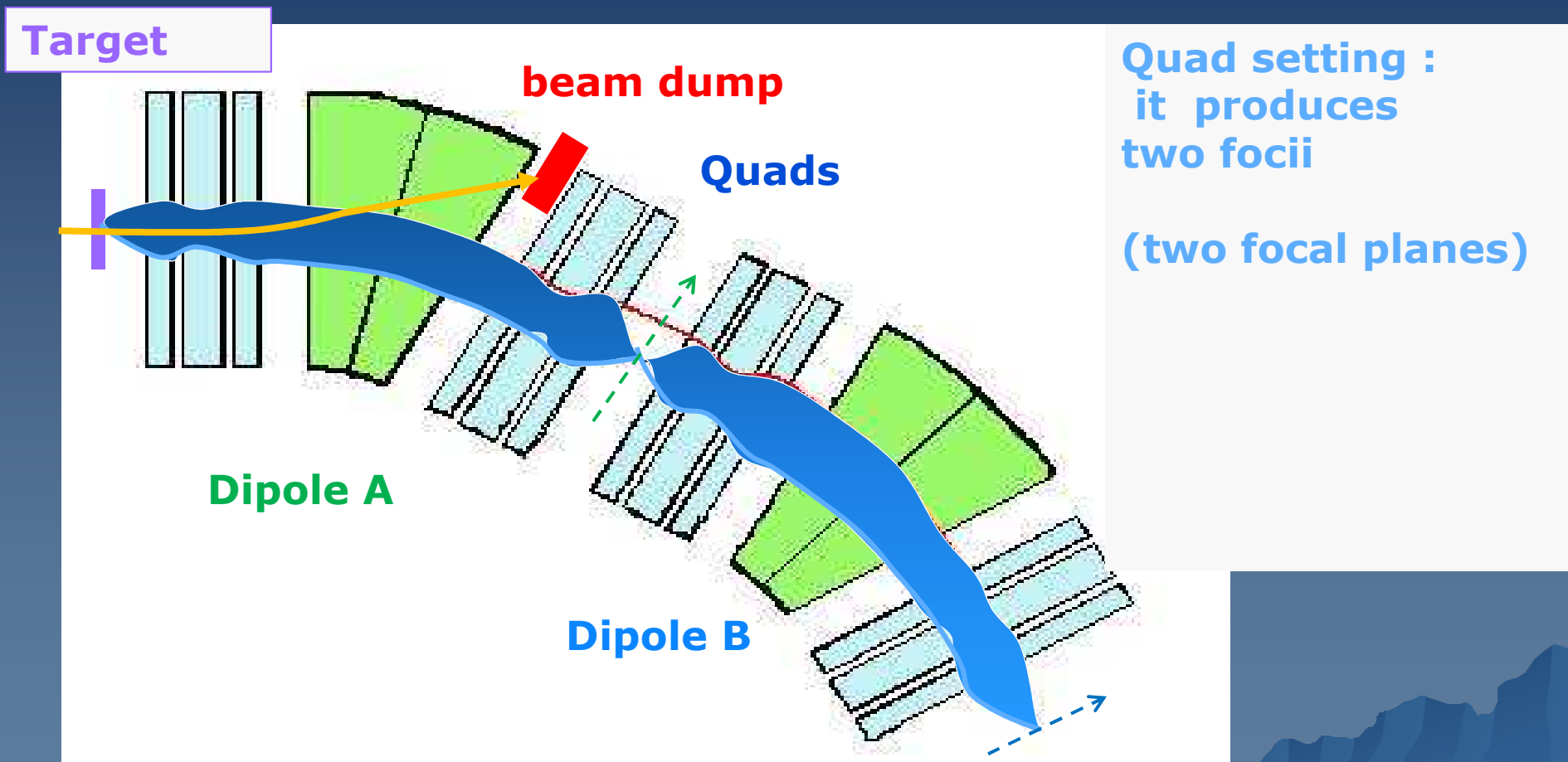


**Purification
to focus on
A specific isotope**

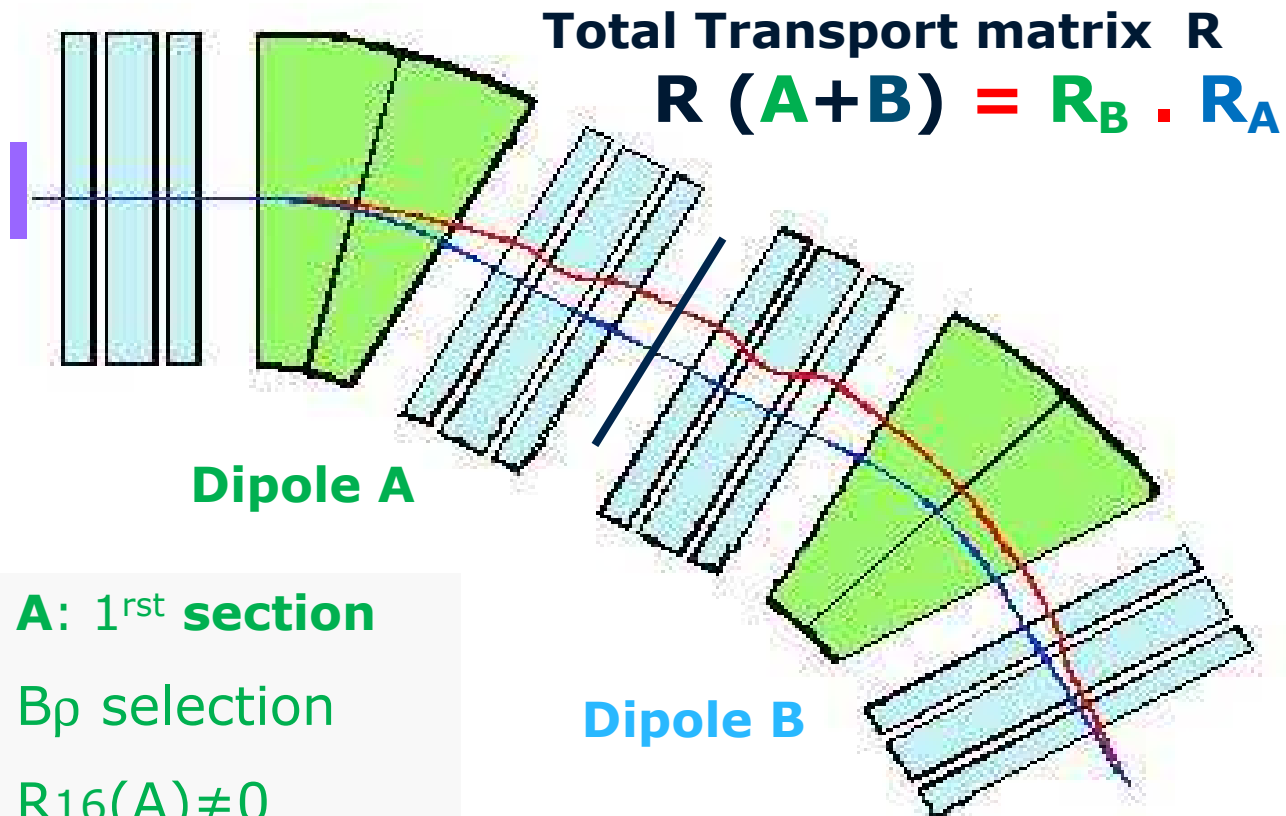
**Measurement :
(Mass; gamma spectroscopy...)**

Fragment separator

2 symmetric sections : A & B



Fragment separator : 2 symmetric sections



A: 1st section

B_p selection

$R_{16}(A) \neq 0$

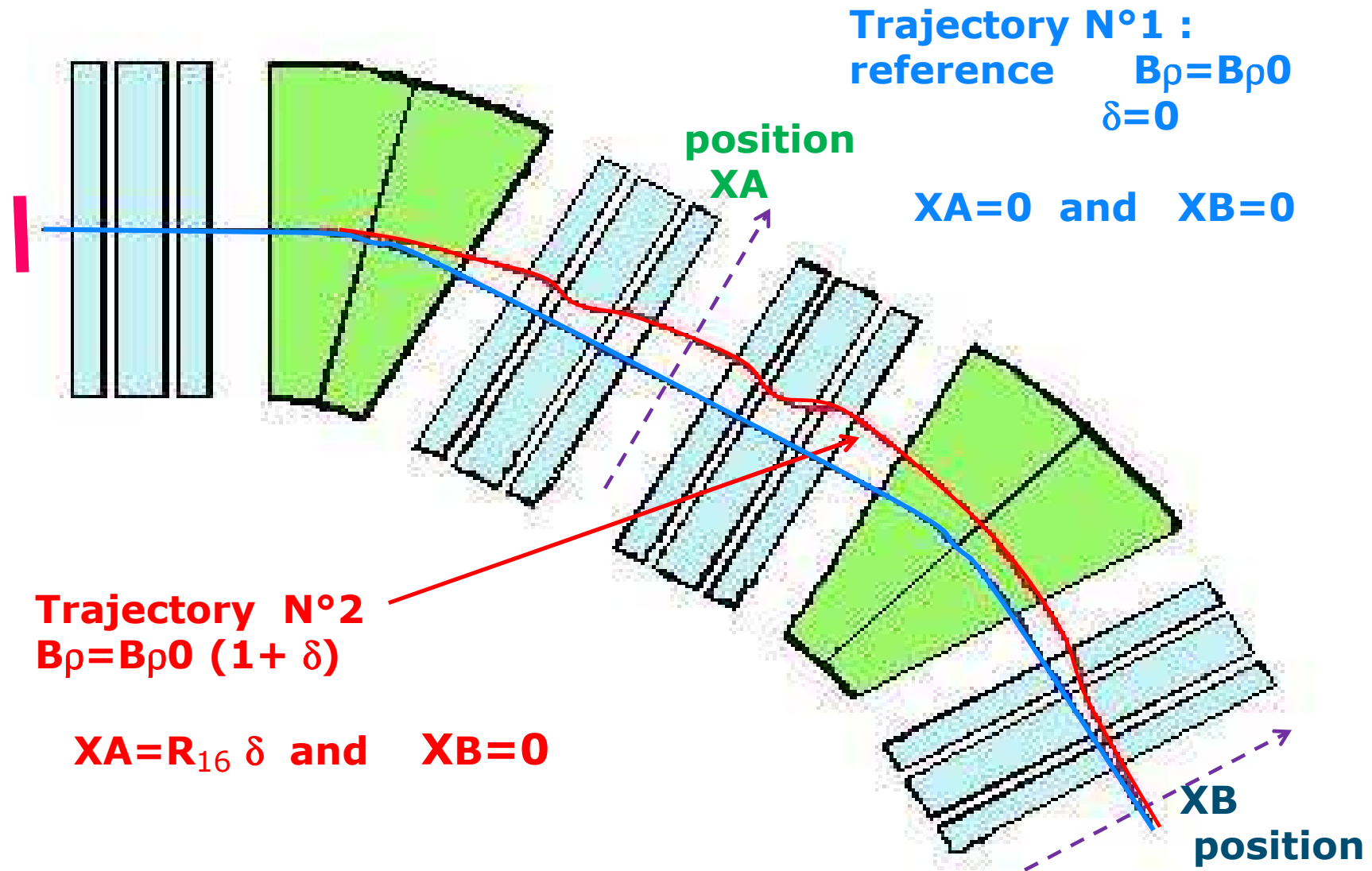
Dipole B

B: 2nd section

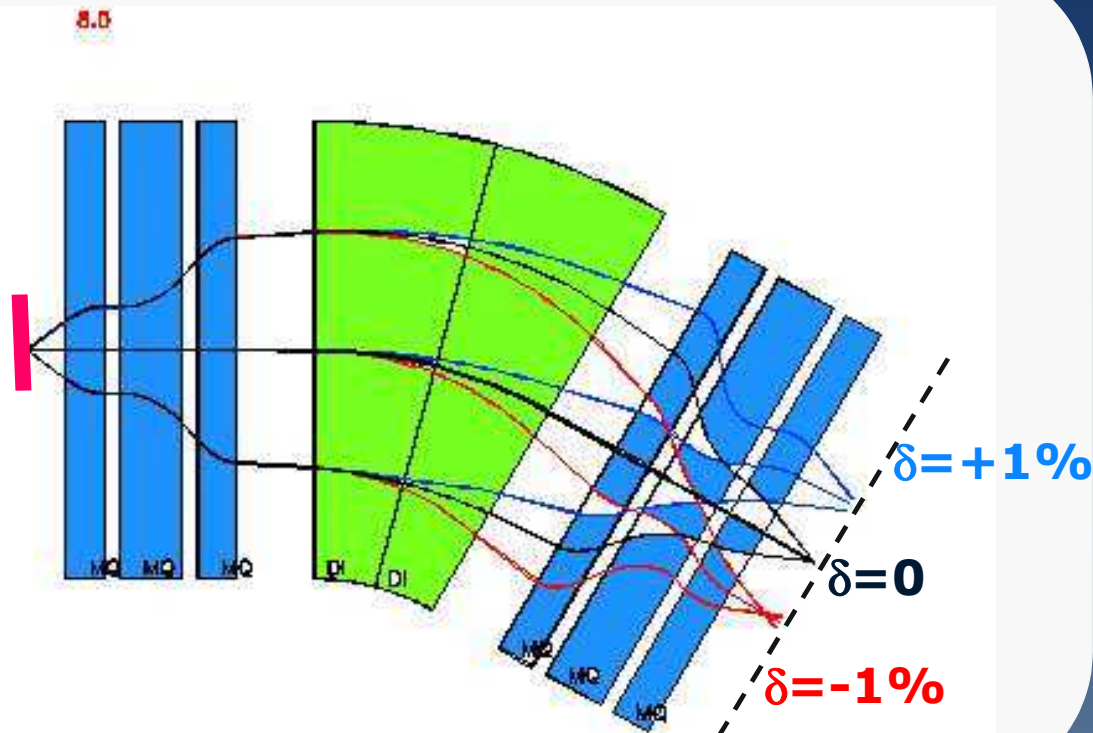
B_p compensation

$R_{16}(A+B) = 0$ (achromatic)

2 Trajectories in a Fragments separator



Fragments separators : dispersiv section optics



Section A :

Focusing

$R_{12} = 0$ (Horizontal)

$R_{34} = 0$ (Vertical)

dispersion

$R_{16}(A) \neq 0$

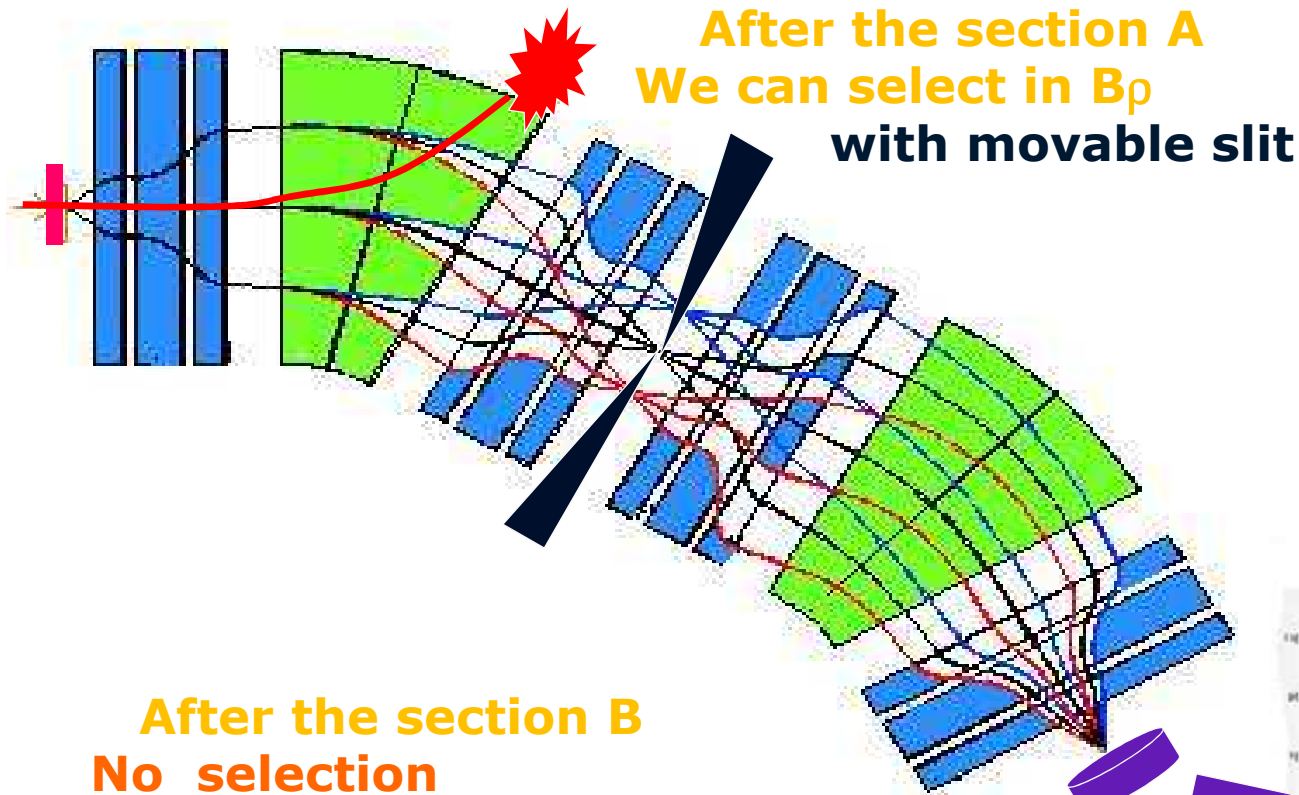
$$Rmatrix(A) = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & 0 & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dispersiv focal plan

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

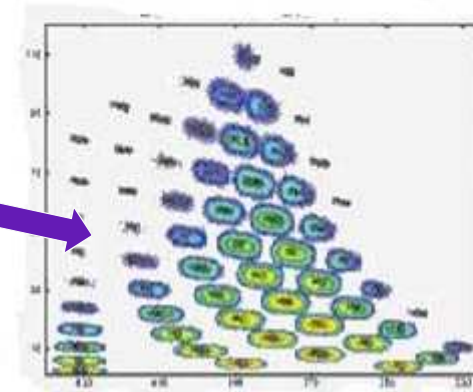
$$B\rho_0 = B_{dipole} \cdot R_{dipole}$$

1 Selection in Fragments separators is not sufficient



$B\rho$ Selection
Is not
good enough

$B\rho$ selection
identification $\Delta E/$
TOF



Primary beam is eliminated, but
Too Many isotopes ($\neq Z$)
produced by fragmentation
are transported up to the end

Magnetic separator with degrador increase the purification (Z dependance)

We consider 2 isobares ($A=34, Z=14$) ($A=34, Z=15$) with same $B\rho$

$B\rho$ selection is
independant from Z

$$B\rho = \gamma M v / Q$$

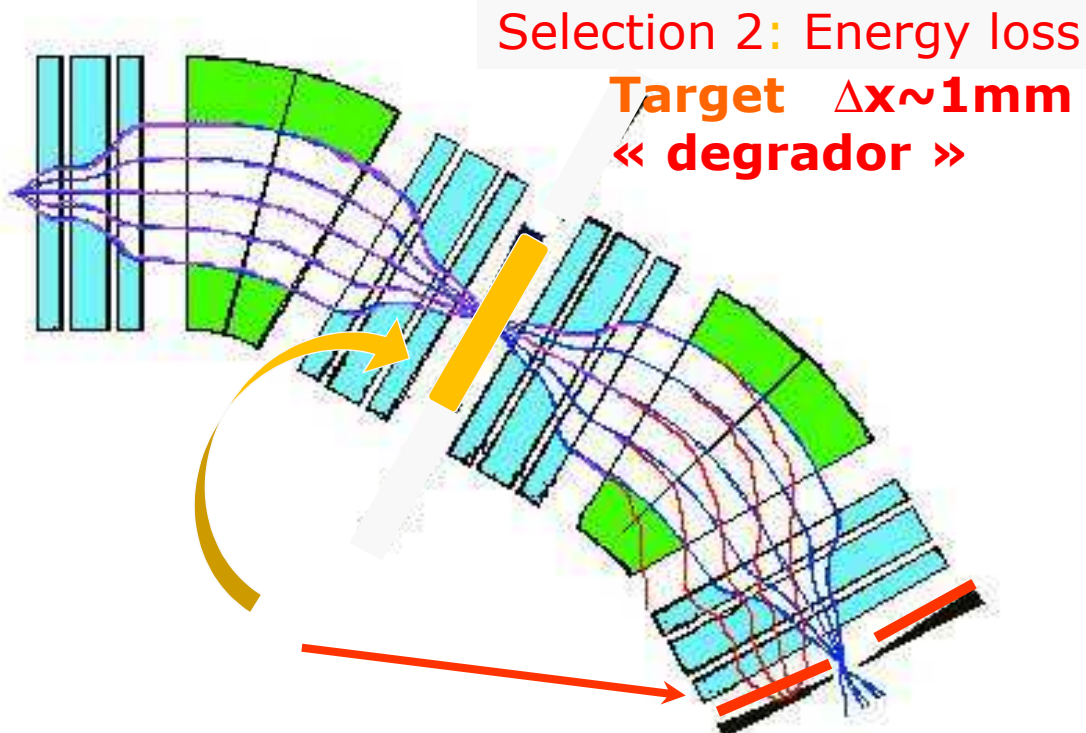
in a target Energy loss
is

« Z dependant »

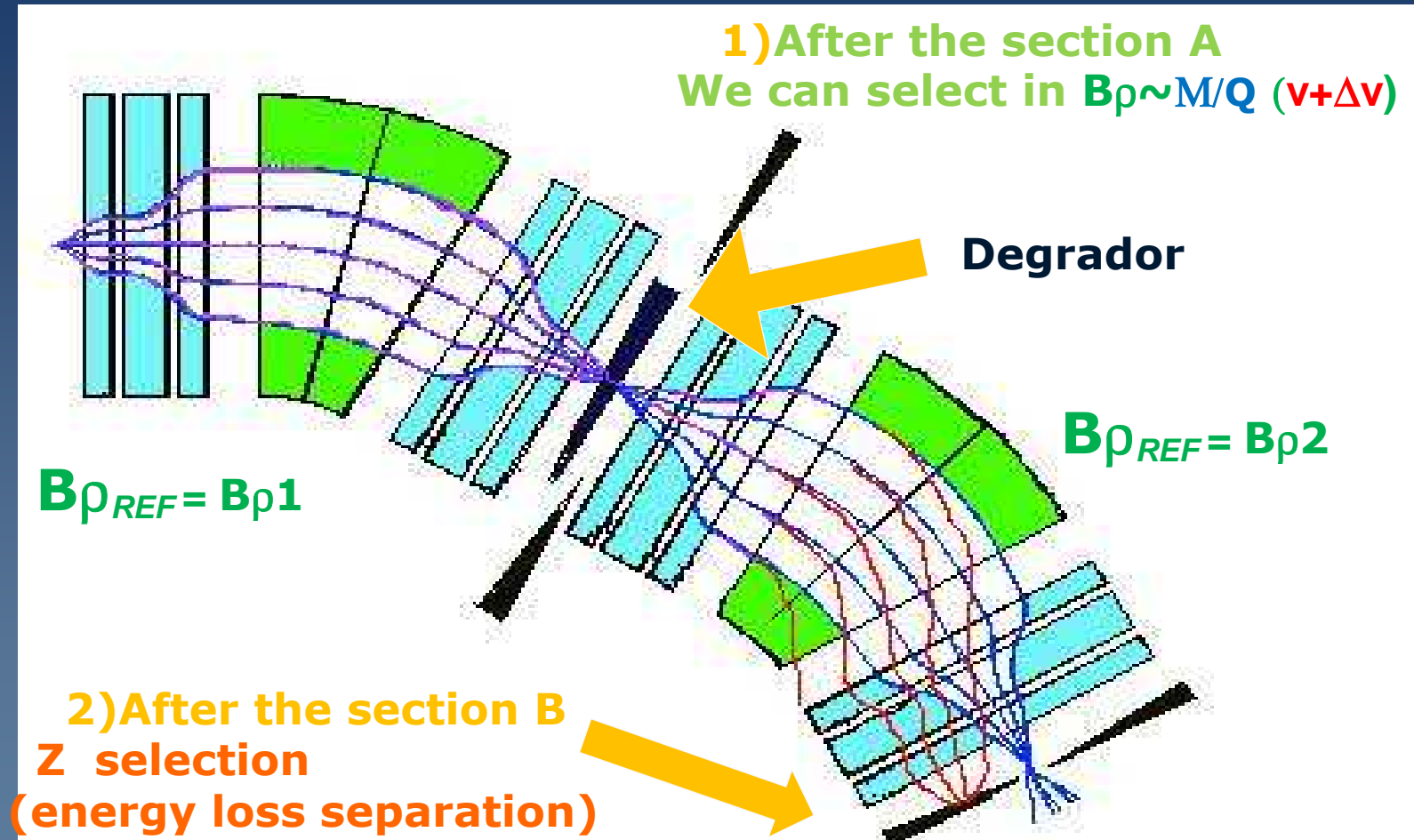
Bethe-Bloch
formula

$$\Delta E = k Z^2/A * \Delta x$$

59



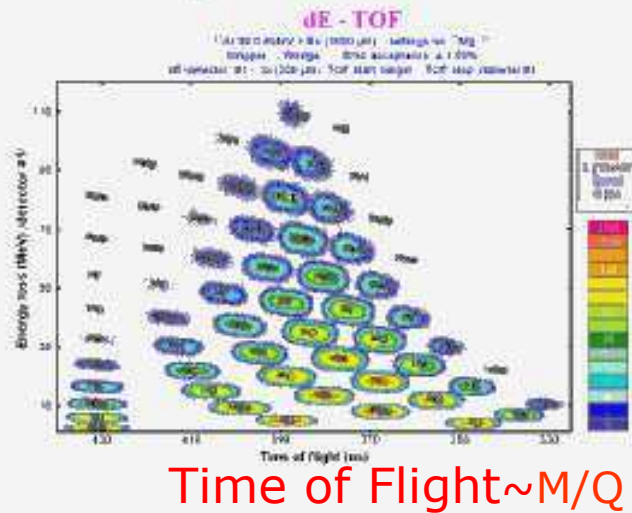
2 Selections in Fragments separators $B\rho + Z$ (degrador)



Selection in Fragments separators & identification

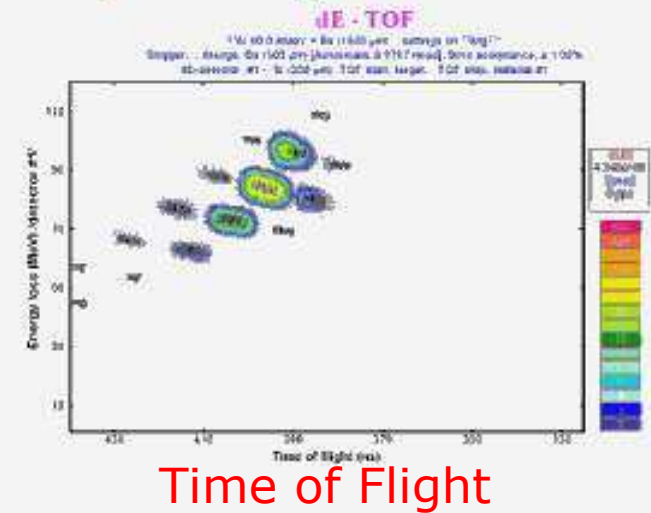
$B\rho$ selection

ΔE
 $\sim Z$

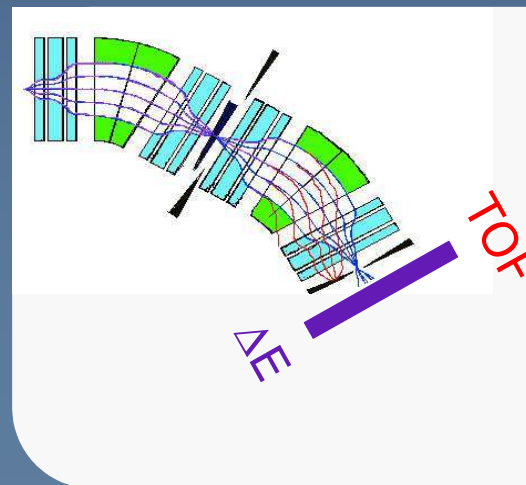


$B\rho$ + degrador selection

ΔE

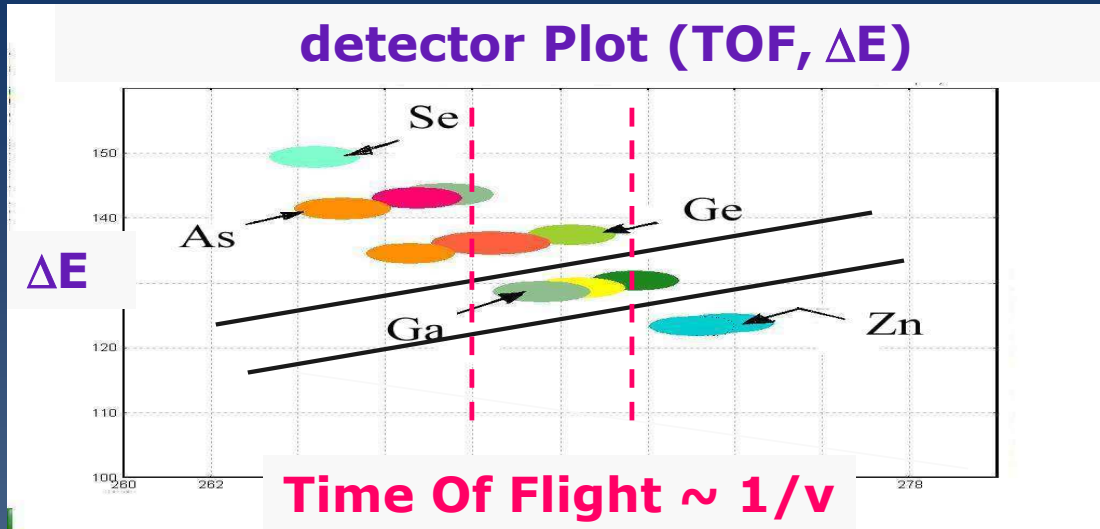


Detector :
Thin Silicone



2 selections
Is much
better for
purity

Often, Isotopes are not well identified ($\Delta E, \text{TOF}$)



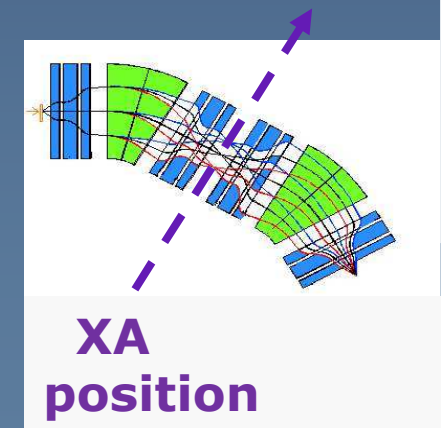
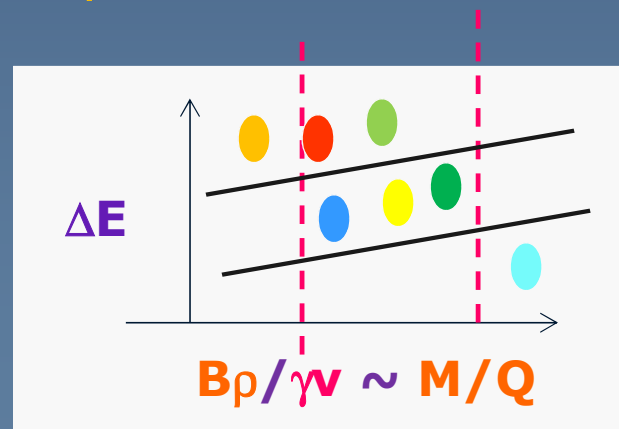
Isotopes
are **mixed** in TOF
Large velocity
distribution Δv

Solution : Measure X_A for each ion : $B\rho = B\rho_0 (1 + X_A/R16)$

The two measurements (TOF, $B\rho$) \Rightarrow give M/Q

$$v = \text{TOF} / L$$

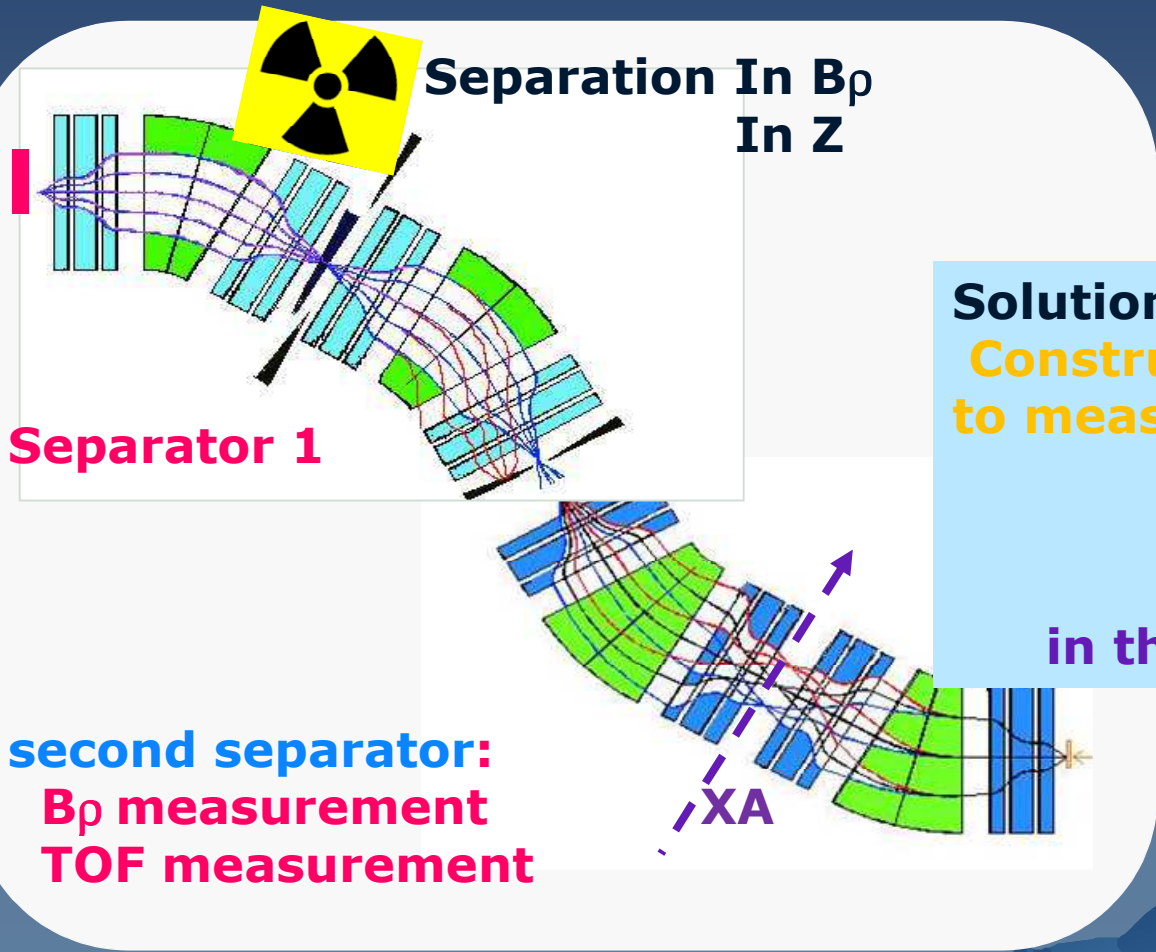
$$M/Q = B\rho / (v \cdot \gamma)$$



Isotopes are not well identified with ($\Delta E, \text{ToF}$)

Install a Detector position for XA : $B\rho = B\rho_0 (1 + XA/R16)$

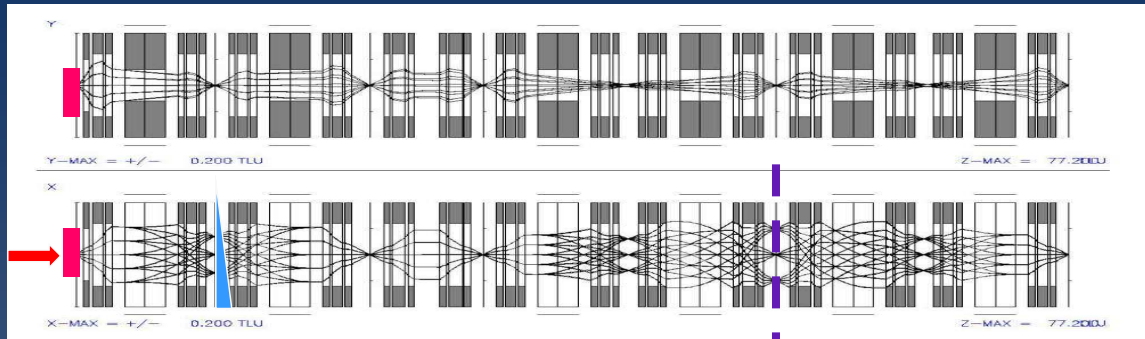
NOT POSSIBLE
(too much intensity before Z selection) 



Solution chosen in BigRIPS :
Construct an additiv spectrometer
to measure $B\rho$ (€ !!)

Install the
position Detector
in the second separator

1 exemple :BIG RIPS (Tokyo)



Specifications

$$L=77\text{m}$$

$$B\rho_{\text{max}} = 7 \text{ Tm}$$

$$\Delta p/p = \pm 3\%$$

$$\Delta\theta = \Delta x' = \pm 50\text{mrad}$$

$$\Delta\phi = \Delta y' = \pm 60\text{mrad}$$

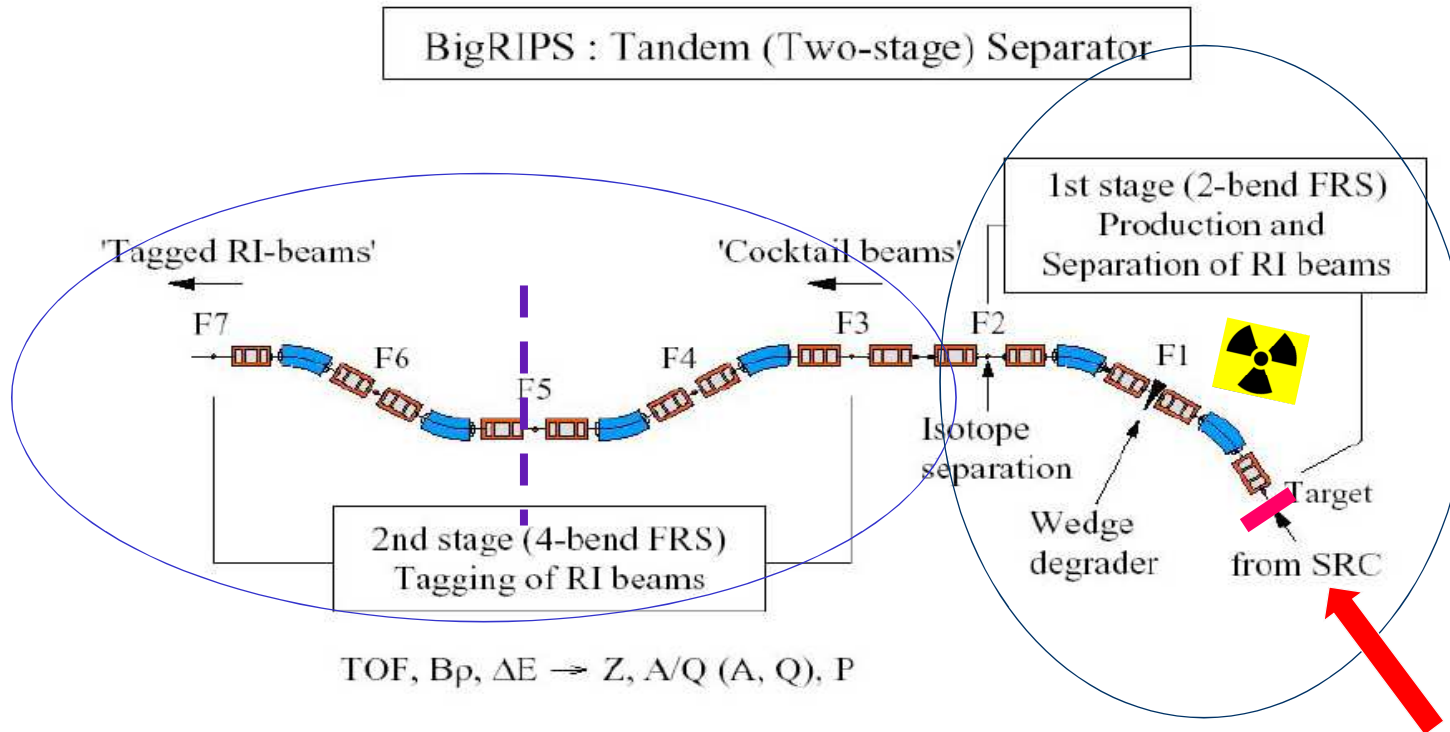


Fig. 2. A schematic diagram of the RI-beam tagging in the BigRIPS separator.

BIG RIPS (Riken) quads

Beam very rigid : $B\rho = \gamma m\mathbf{v}/Q = 7 \text{ T}\cdot\text{m}$ (Beam **300MeV/A**)
with **high v** !

Super-ferric quadrupole triplet :

Very strong focusing : supraconducting coils (NbTi), with pole (Fe)

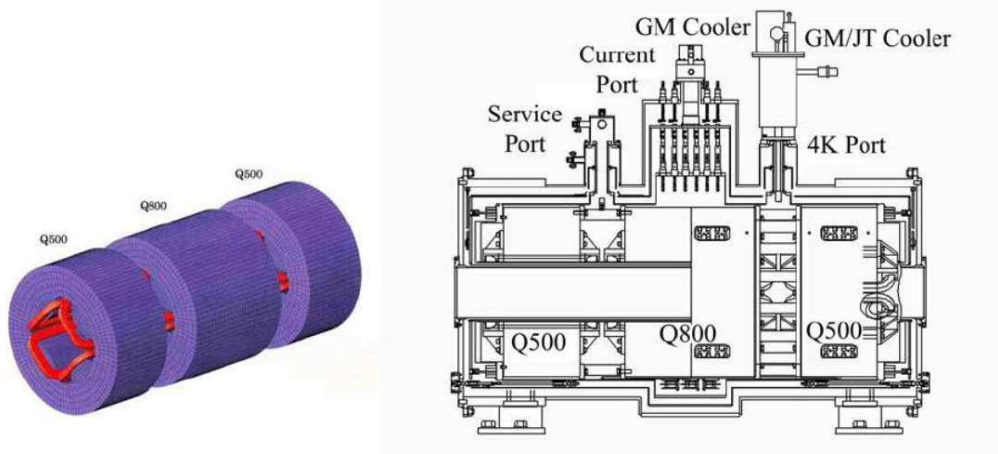


Figure 22: Schematic view of the RIKEN prototype quadrupole triplet (left side) and its installation into the cryostat (right side) [24].

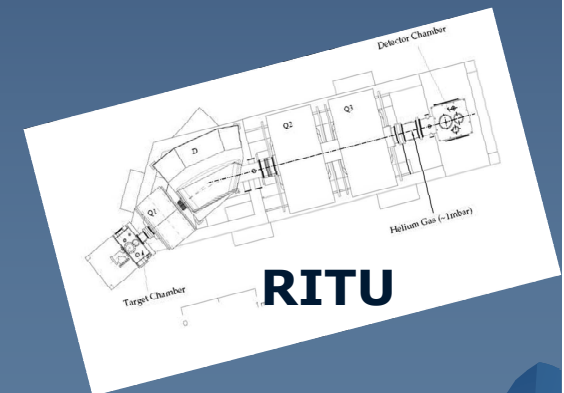
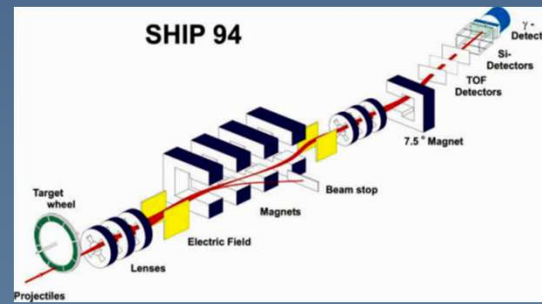
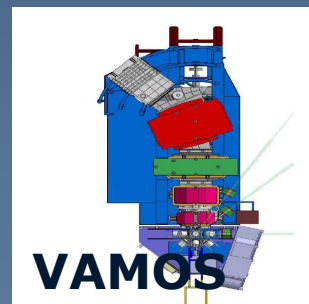
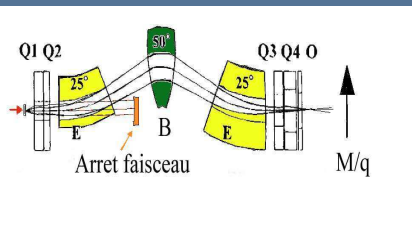
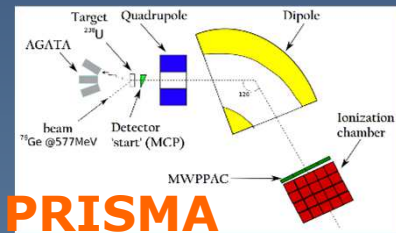
- Supra-conducting coils (i very large, B close to saturation)
- Raperture very large = 0.1m ; Bpole-max# 2 Teslas
- GradientMax=2T/0.1m=20.T/m

« Recoil » spectrometer : nuclear physics at low energy (1-10MeV/A)

Many experimental problems => A large variety of devices
Reactions : fusion-evaporation, transfer,..

Goals :

- 1) Very efficient primary beam suppression
- 2) Help identification



SPECTROMETER TUNING AND DIAGNOSTICS

Tuning rely on - **B field** measurement
- Beam measurement

Beam Diagnostics : dedicated Robust detectors for
beam tuning

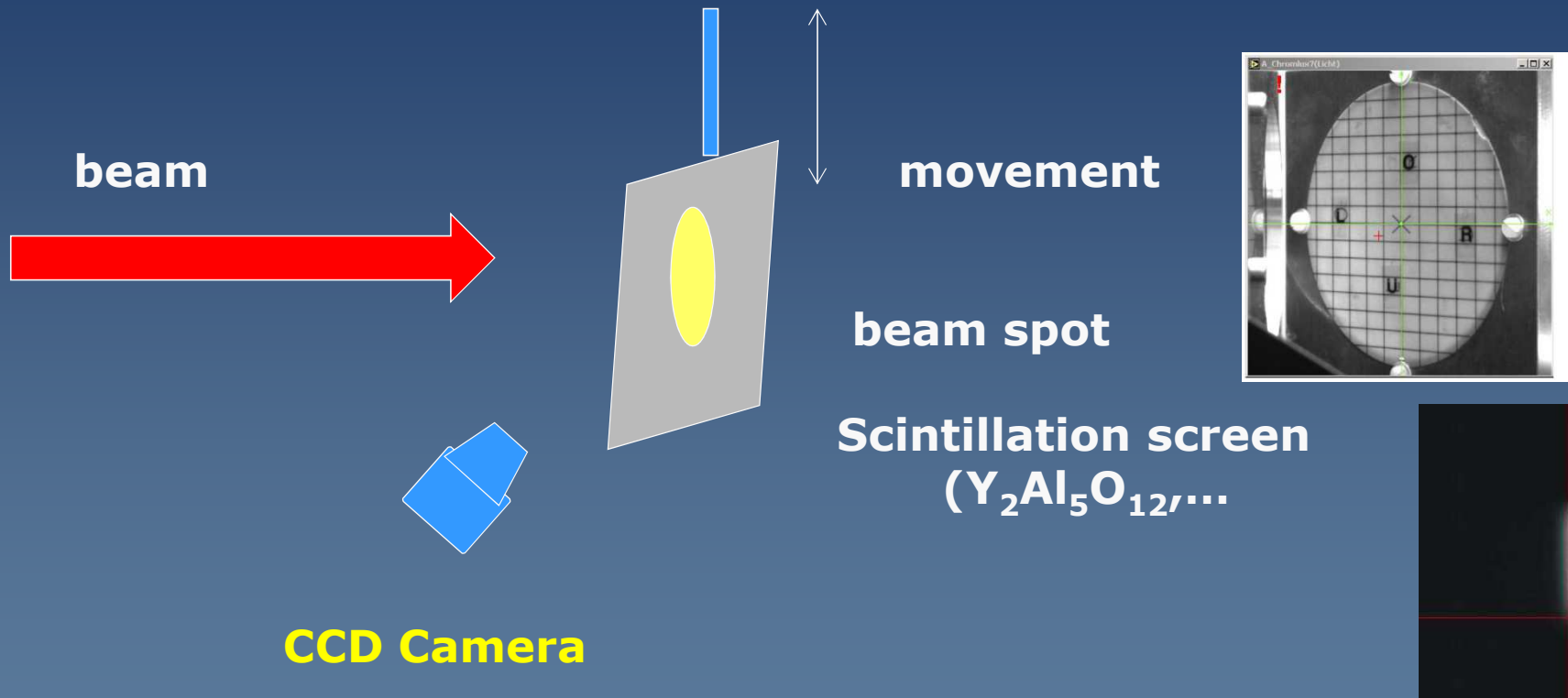
Statistical information on the beam $(\bar{X}, \sigma_x, \sigma_T, \langle I \rangle \dots)$

1rst step : check the primary beam

- profil measurement (alignement, focus)
- intensity check

SPECTROMETER TUNING

Beam diagnostics : scintillator screen



Relatively low cost , but

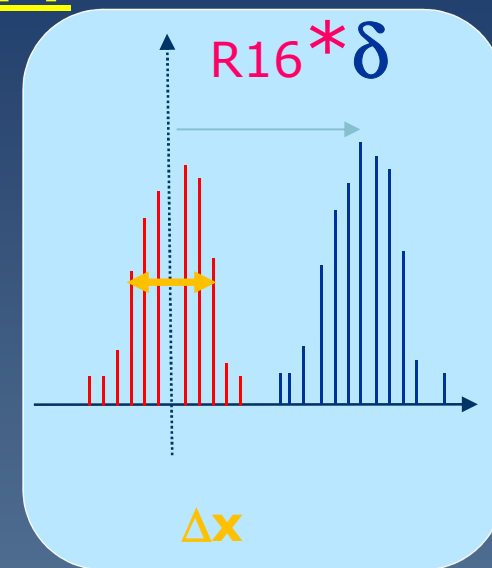
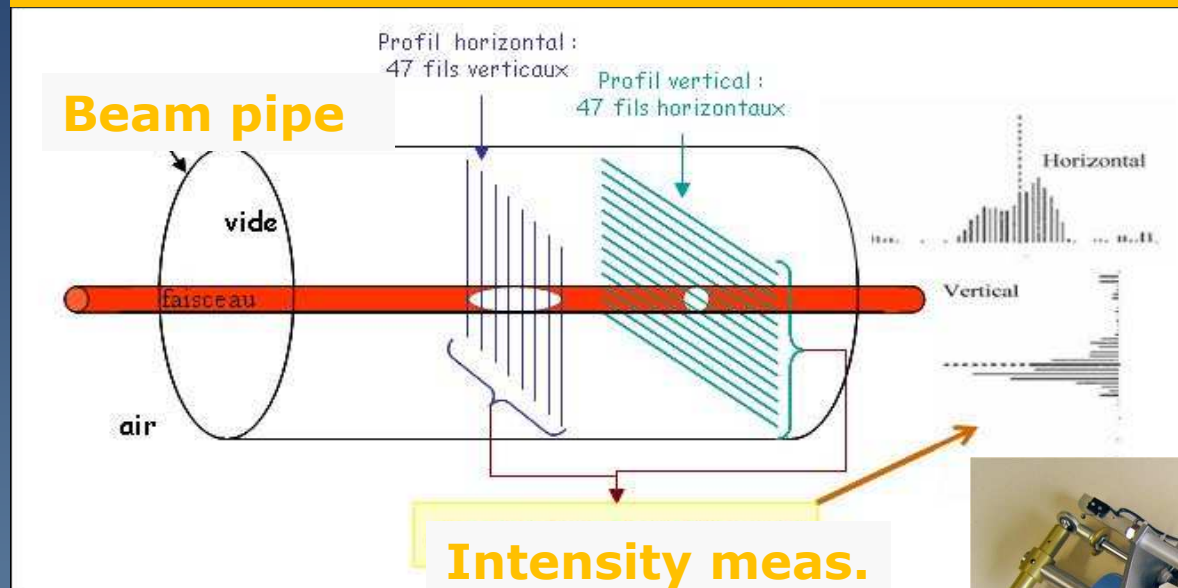
- only 1 profil measurement
- not very precise

SPECTROMETER TUNING

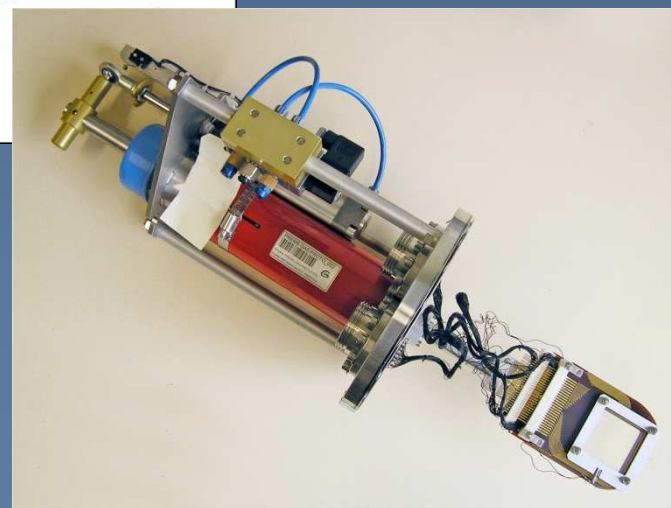
Beam diagnostics : profil monitors

Reconstruct the beam intensity in X and Y

Profil monitor : HORIZ. and VERT. wire



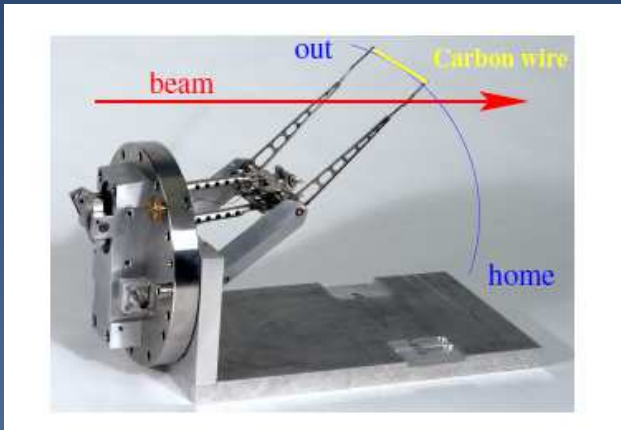
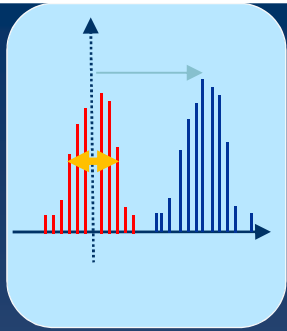
Usefull for **beam alignment**
focusing check
R16 measurement



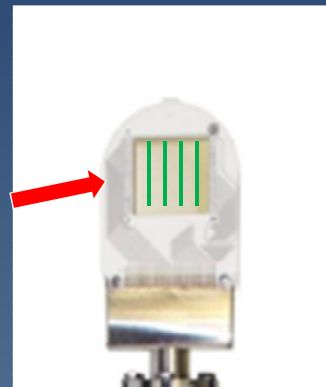
SPECTROMETER TUNING

Many Profil monitors

for different beam intensities



Rotating wire
i# 10^{12-14} pps
(Cern)



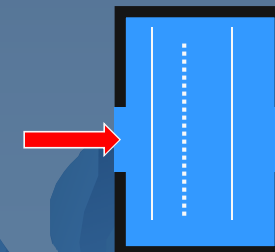
Wires
i# 10^9-11 pps

(Ganil)

« Gas Profil »
i# 10^3-7 pps

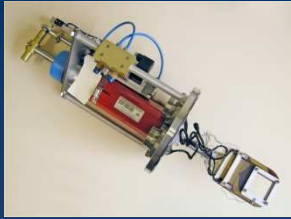


Gas ArCO₂ +HV



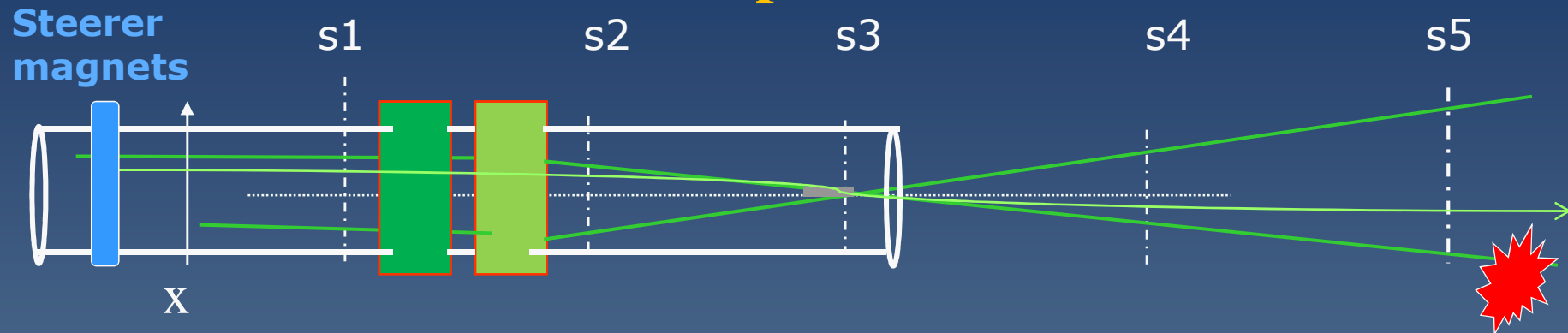
Proportional counter

Specific technologies
adapted for \neq (intensities, Energies)

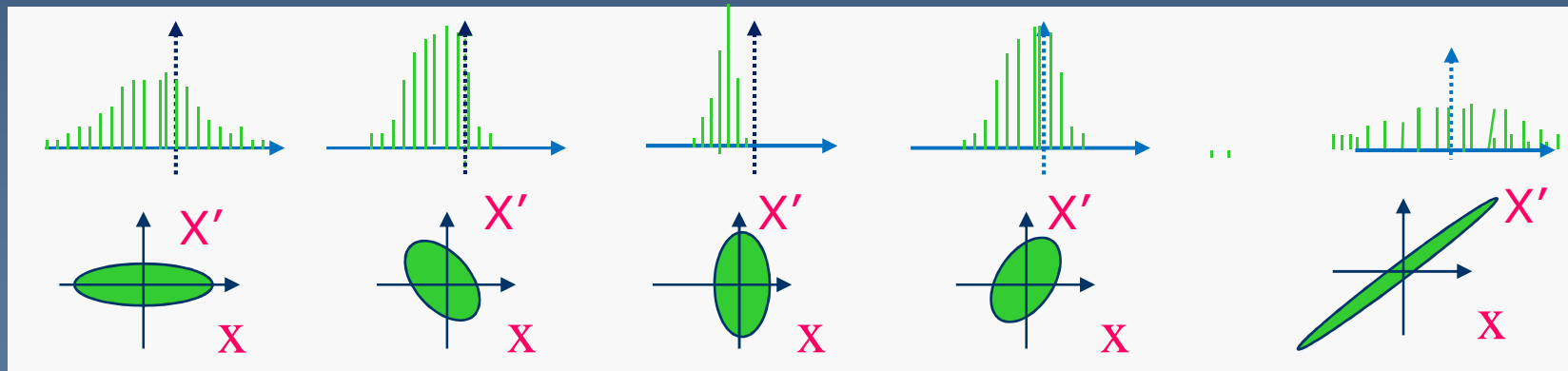


TUNING

Checking size and alignment with profil monitors



Beam
Profil
monitor



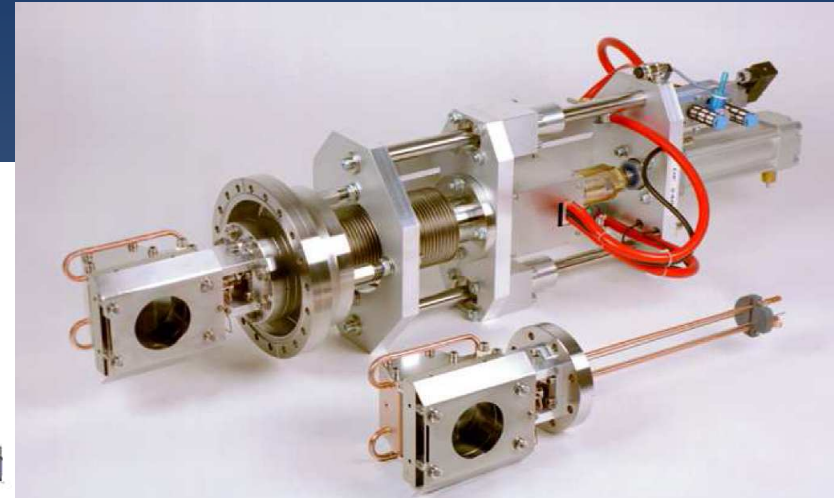
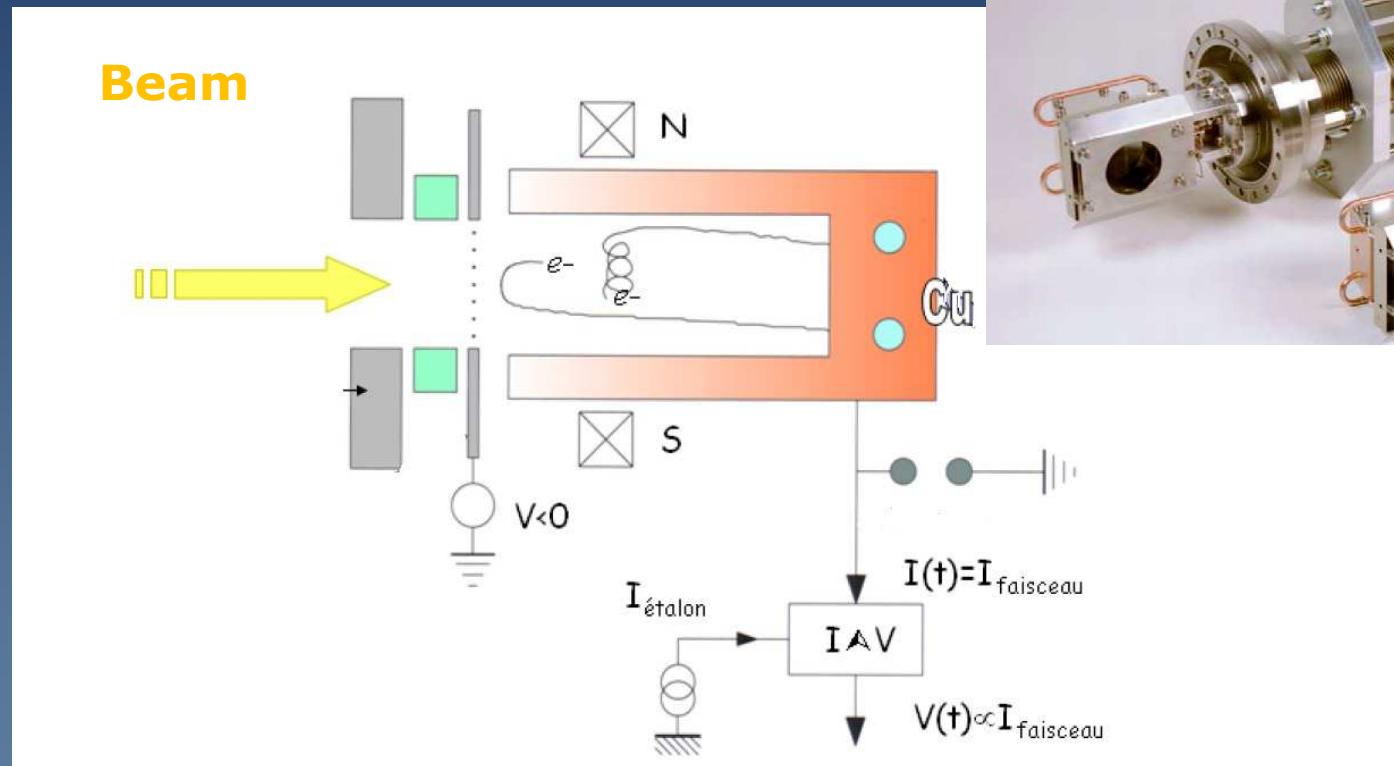
ellipsoid area = $\pi \Delta x \cdot \Delta x' = \text{Emittance}$

Emittance = constant if Energy = constant

SPECTROMETER TUNING : check the intensity

Beam diagnostics : **Faraday cup**

Intensity measurement



Particle per second

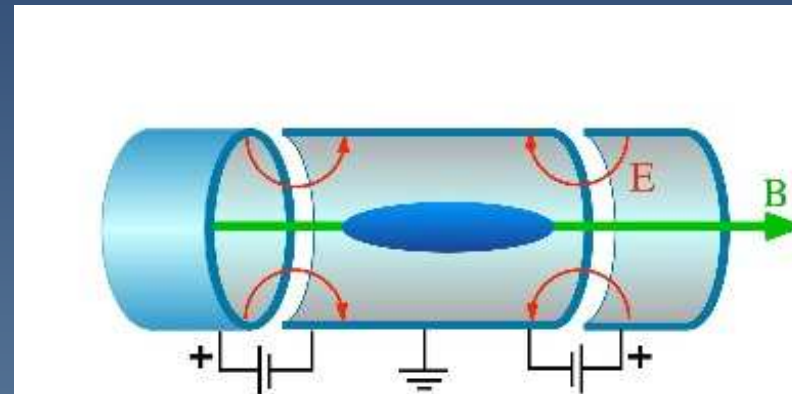
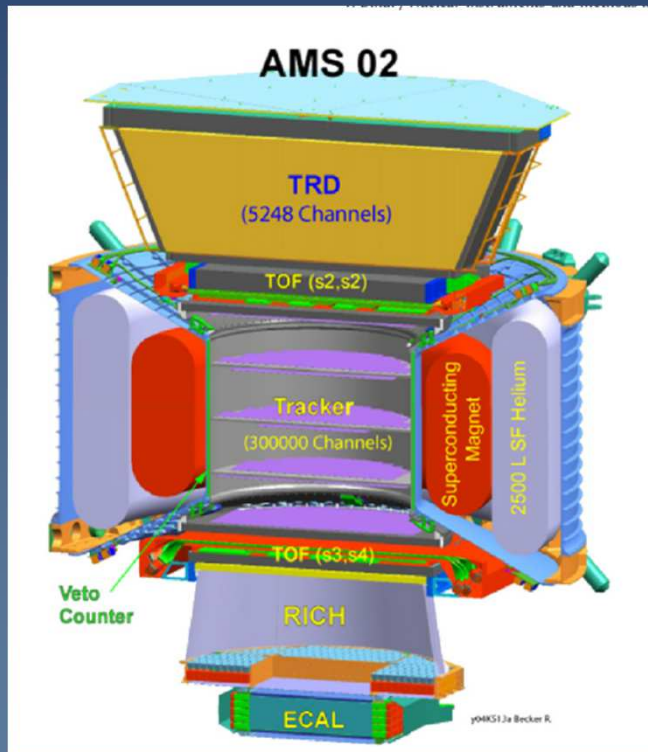
$$N_{pps} = IA/Qe$$

$$= I_{\mu A} 10^6 / [Q 1.6 \cdot 10^{-19}]$$

Summary

- I) History and evolution of the spectrometer
- II) Magnetics spectro/separators with accelerator's beams
technical device : quad, dipole
Beam optics concept
- III) Spectrometers without accelerator
 - 1 exemple for Astroparticle
 - Penning Traps

III) Spectrometer experiments without accelerators



Penning Traps

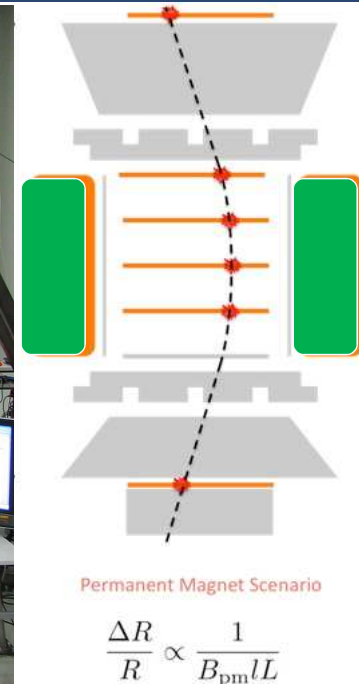
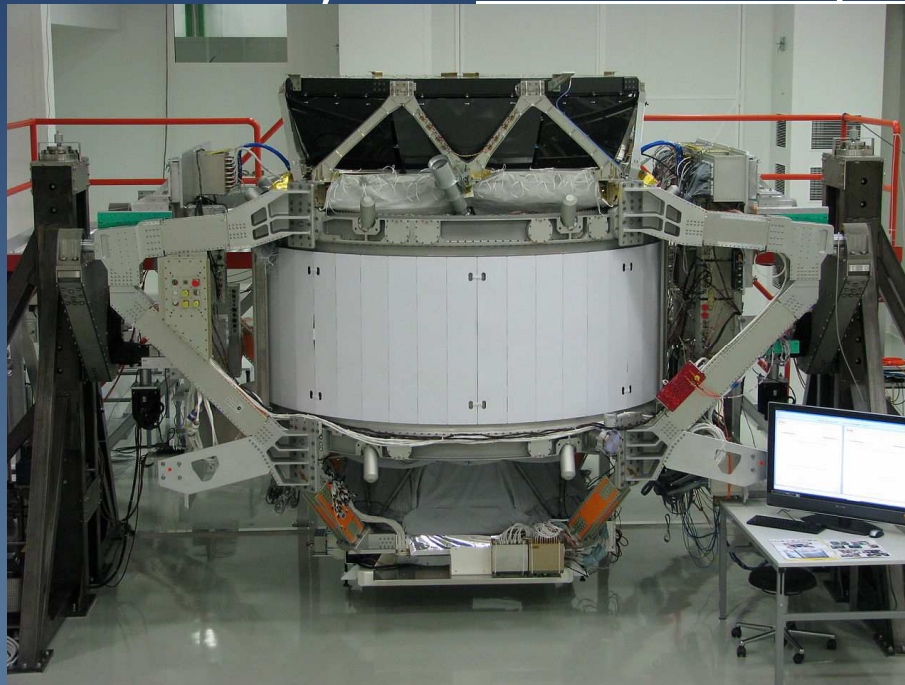
1eV ions

AMS at ISS (10-300GeV e⁻)

AMS :a Spectrometer in space

7 tons : 1 dipole magnet + trackers+ 1Calorimeter

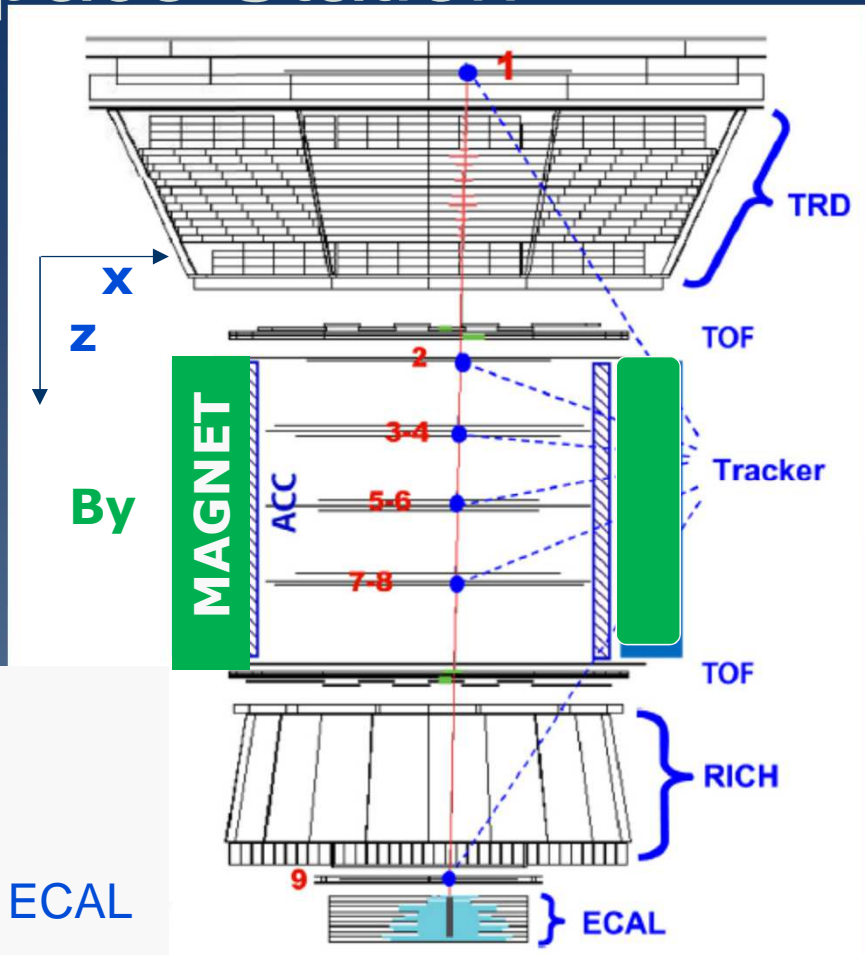
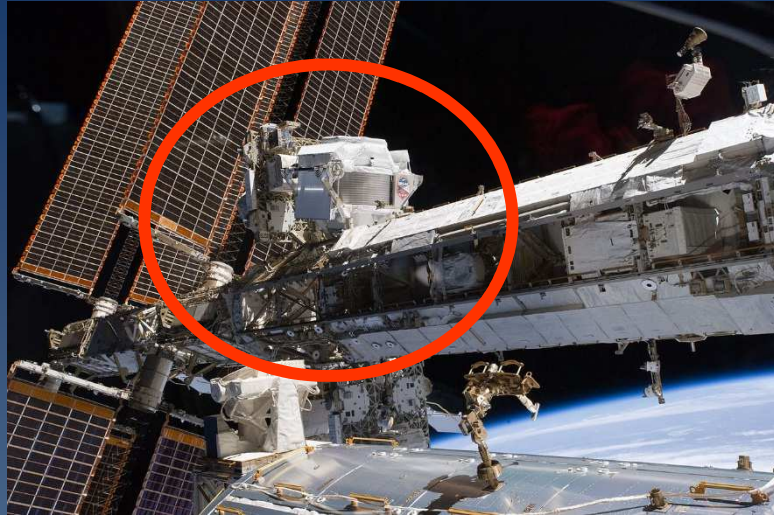
$P=2,3 \text{ kW}$ $S_{\text{active}}=0,8 \text{ m}^2$



Cosmic Ray Measurement + quantification of Matter/antimatter

$e^- // e^+$; $p // \bar{p}$; heavy ions He; Li;....

AMS2 at Int. Space Station



The reconstruction of a 300 GeV electron measured by AMS2,
with the signals in TOF, tracker, RICH and ECAL

- Spectrometer+detector that measures antimatter
in cosmic rays : $e^- // e^+$ [10-300 GeV]
 $p // pbar$

AMS2 with « a dipolar field » $B=B_x$

higher field=higher dispersion= better Resolution

Electromagnet excluded : too heavy//high power

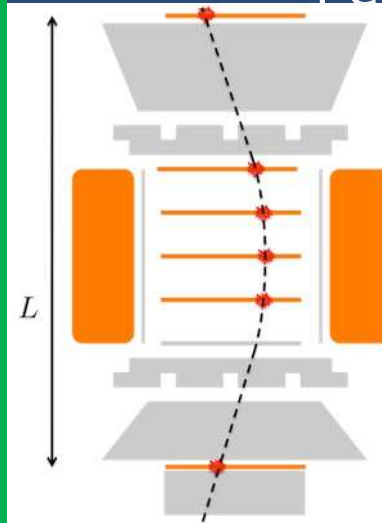
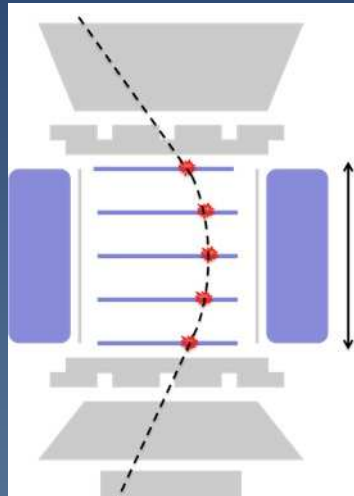
So 2 magnet options : sufficient field ? & uniformity ?

Superconducting coils

$B_z \sim 0,8 \text{ T}$

B_x Produces
by current distribution

$$I(\theta) \propto I_0 \cos(\theta)$$

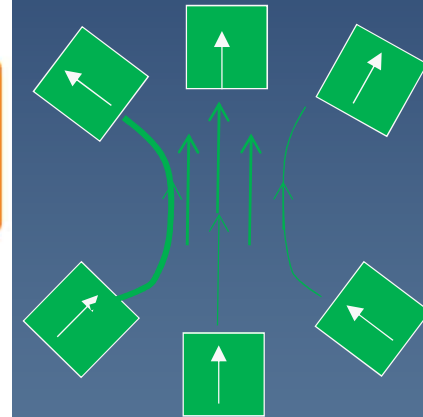


Permanent Magnet Scenario

$$\frac{\Delta R}{R} \propto \frac{1}{B_{pm} l L}$$

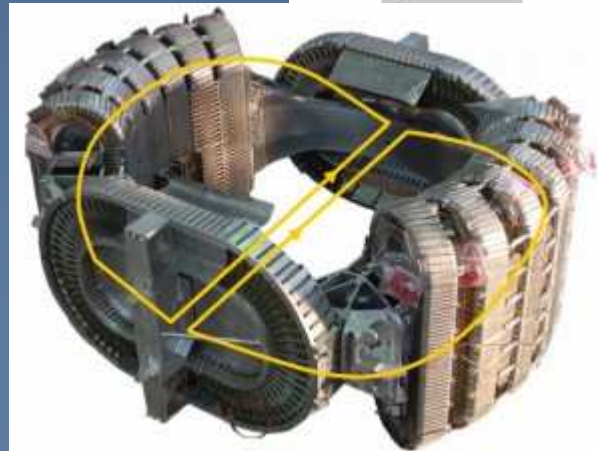
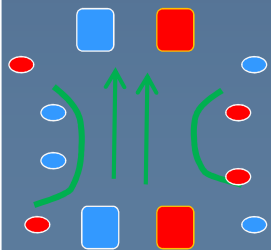
Permanent magnets

$B_z \sim 0,15 \text{ T}$



B_x Produces by magnetization
distribution

$$M(\theta) = M_0 \cos(\theta)$$



Rmatrix for a magnet (ϕ , R)
 very convenient for the tracking in a spectro

$$\begin{aligned} \mathbf{X}_{\text{final}} &= -\mathbf{R}_{11} \mathbf{x}_0 + \mathbf{R}_{12} \mathbf{X}_0' + \mathbf{R}_{16} (\mathbf{B}_{\rho 0} - \mathbf{B}_{\rho 0}) / \mathbf{B}_{\rho 0} \\ &= -\cos(\phi) \mathbf{x}_0 + R_0 \sin(\phi) \mathbf{X}_0' + R_0 (1 - \cos(\phi)) \delta \end{aligned}$$

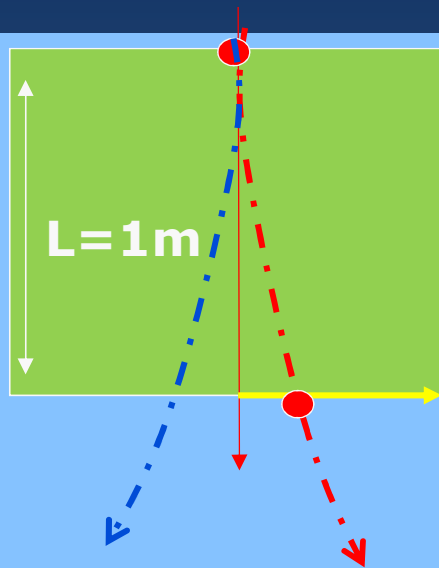
Rdipole=

$$\begin{pmatrix} \cos \phi & R \sin \phi & 0 & 0 & 0 & R(1 - \cos \phi) \\ -1/R \cdot \sin \phi & \cos \phi & 0 & 0 & 0 & \sin \phi \\ 0 & 0 & 1 & R\phi & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin \phi & -R(1 - \cos \phi) & 0 & 0 & 1 & R\phi/\gamma^2 - R(\phi - \sin \phi) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Transport matrix
 For a dipole with
 angle ϕ

AMS2 : Tracking in a dipole magnet

1rst order



Reference particules = e- 50GeV

$$e^- (50\text{GeV}) = B\rho_0 = 167 \text{ Tm}$$

$$\theta = s / R_0$$

$$R_0 = 167\text{T.m} / 0,8\text{T} = 200\text{m}$$

$B_y = 0,8\text{T}$

Position of 50geV e- : $X_{f0} = R_0 (1 - \cos(\theta)) = L^2 / 2 R_0 = 2,5 \text{ mm}$

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

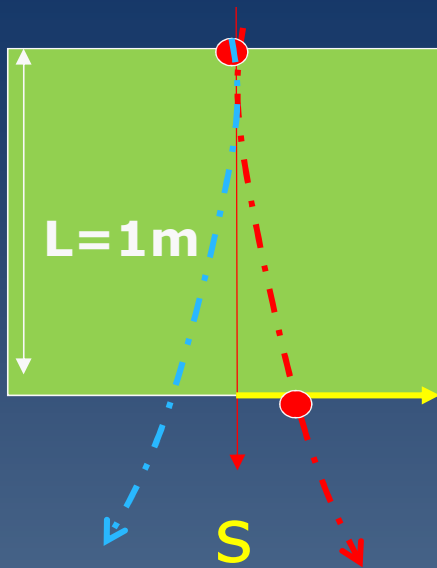
$$X_{\text{final}} = -\cos(\theta) x_0 + R_0 \sin(\theta) X_0' + R_0(1 - \cos(\theta)) \delta$$

(= see transport matrix of a dipole matrix)

So $X(s=L) = ?$

$$L^2 / 2 R_0 + X_0 + R_0 \sin(\theta) x_0' + R_0 [1 - \cos(\theta)] \delta$$

AMS2 3 unknown (m, Z, E)



Tracking e- in dipole magnet 1st order

Reference particles = e- 50GeV

$$\delta = (B\rho - B\rho_0) / B\rho_0$$

$$X(s) = X_0 + s^2/2 R_0 + S X'_0 + R_0 [1 - \cos(\theta)] \delta$$

Tracker : Fit $X(s)$: find $\delta = (B\rho - B\rho_0) / B\rho_0$

Calorimeter = gives E & RiCh gives β

$$B\rho = P/Q + E = (\gamma - 1) mc^2 + \beta = (m, Z, E)$$

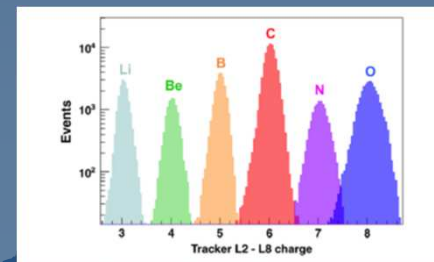
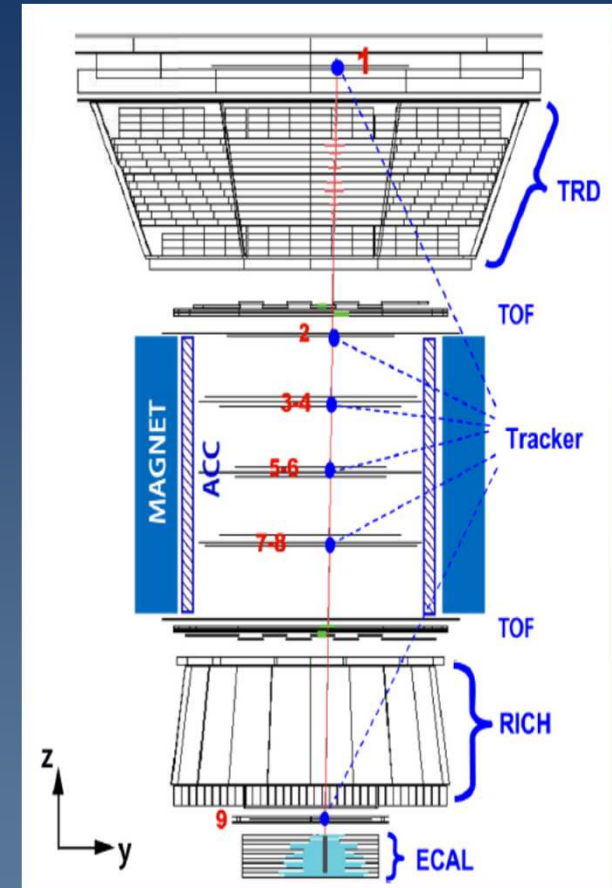
AMS detectors and results

- TRD** : A transition Radiation detector (gas)
Allows the separation of e/p
- Ring Cerenkov detector** : $\Delta\beta/\beta \sim 0,1/Z \%$
z measurement
- TOF detectors** : are a fast trigger
- Trackers** : 10micron for e/p
e: R=2,5% in Brho up to 100GeV

[2011-2015]

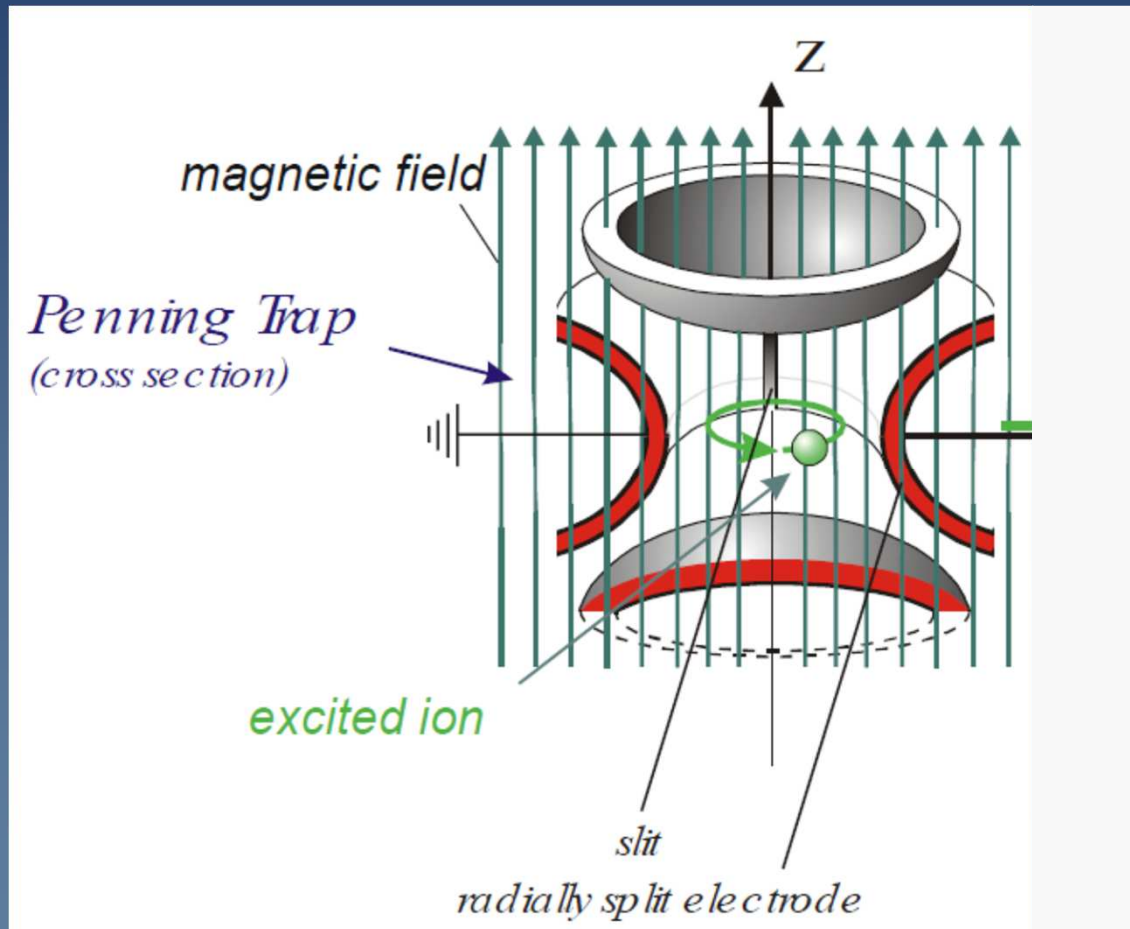
**Some e+ has been detected @100GeV
source of antimatter
in the universe ?!**

Data has been collection for radioprotection
issues (long fligth toward mars ?)
P, He, Li



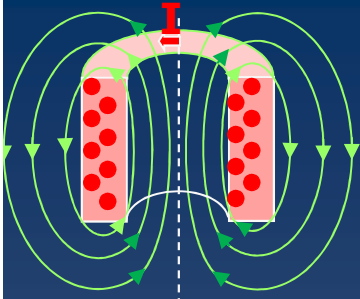
Penning Traps : high resolution mass spectroscopy

Used in reseach but applications expending
The ultimate mass resolution device



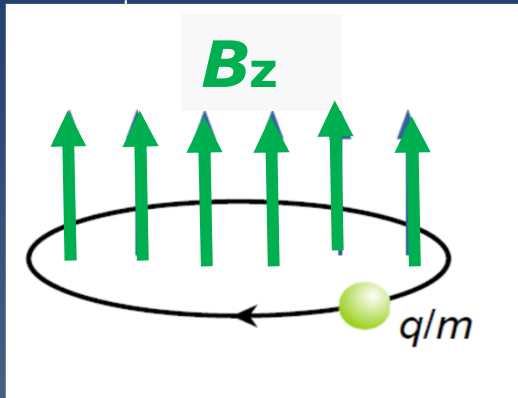
3 tricks

- 1) Confinement ($B_z + E_z$)
static EMfieds
- 2) Excitation (RF field)
- 3) Extraction
(Tof measurement)



Penning Traps (1) : confinement

Solenoid coil : field (B_z) is axial

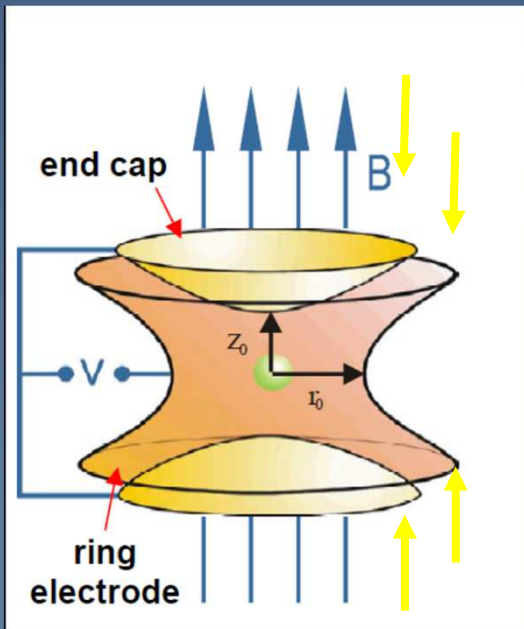


3 D trapping technics (Radial+longitudinal)

1a) Radial (x,y) confinement with B_z

-natural cyclotron frequency

$$\omega_c = \text{frequency} / 2\pi = qB/m$$



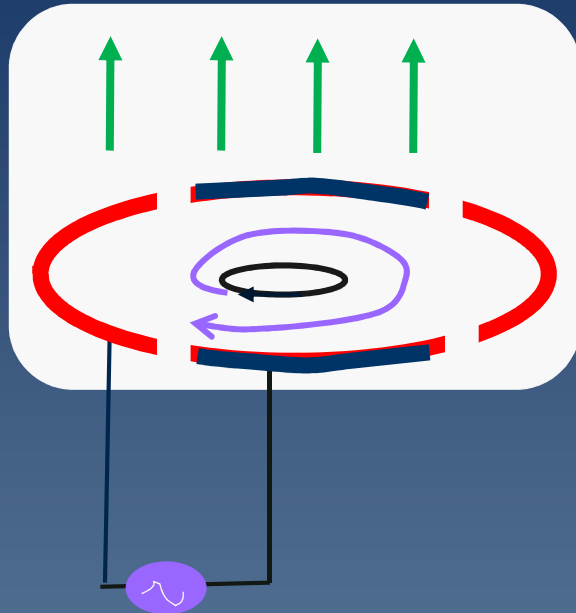
1b) Longitinal (z) confinement with Electric field

$$V = U_0 (2z^2 - x^2 - y^2)$$

$$F_z = -4z U_0$$

hyperbolic field : End caps + Ring electrode

Penning Traps (2) :excitation at resonance



excitation RF : 4electrodes

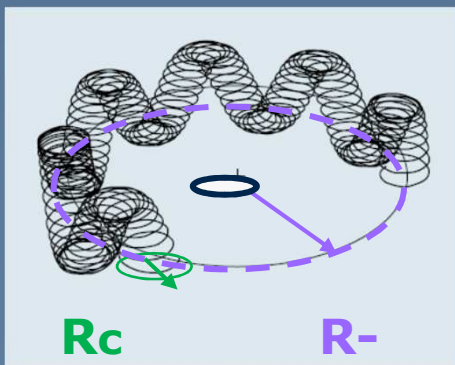
- natural cyclotron frequency

$$\omega_c = f_{\text{revolution}}/2\pi = qB/m$$

-Excitation with RF electric field

$$V(t) = U \cos(\omega_{rf} t)$$

if $\omega_{rf} = \omega_c$ Radial motion increases



in reality, the motion is complex (coupling)
radial (Bz) and axial motion (Fz)

$$\omega_c = \omega^+ + \omega^- \quad \omega_z^2 = 2 \omega^+ \cdot \omega^-$$

$$\omega_c^2 = \omega^{+2} + \omega^{-2} + \omega_z^2$$

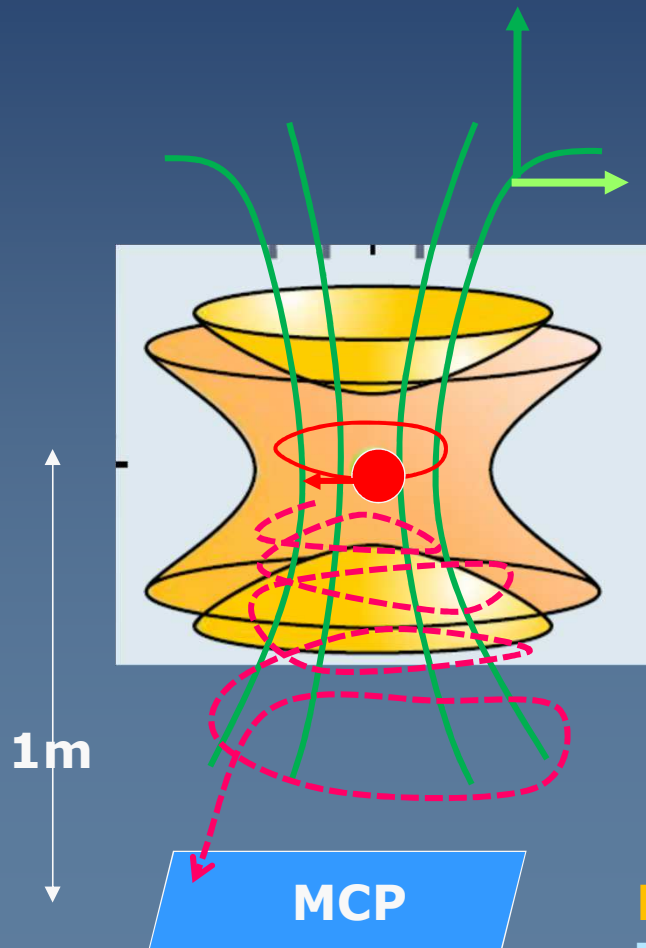
Penning Traps (3) : extraction

Extraction with a time of flight measurement

B_z main Force(radial) = $q \mathbf{v} \times B_z$
:confinement

$$\mathbf{B} = B_z + B_y + B_x$$

B_x



Extraction in solenoid's end fields

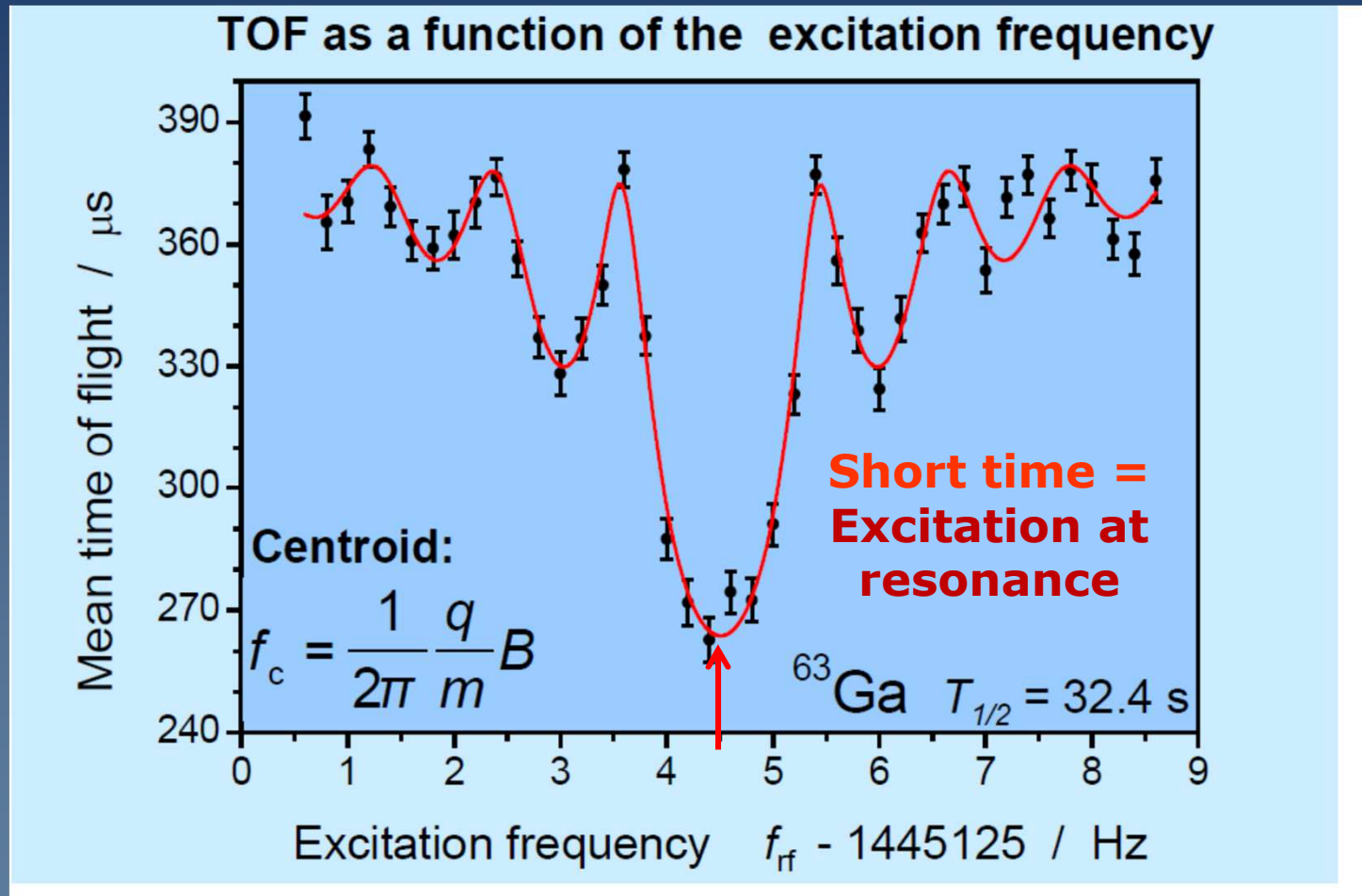
$$\text{Force (z)} = q [v_x B_y - v_y B_x]$$

Radial act on
The Excited ions with large V_{radial}

Excited ions reach the detector (MCP)
TOF Measurement

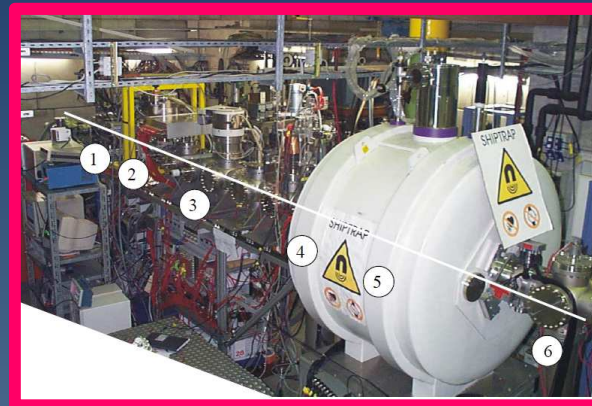
Penning Trap : mass measurement

$$R = \delta m / m \text{ } \#[10^{-5} - 10^{-8}] \sim T_{\text{excitation}} F_{\text{rf}}$$



Penning Traps : Many experiments operated in the world

Magnet = superconducting solenoid (Bz#3-6 Tesla)



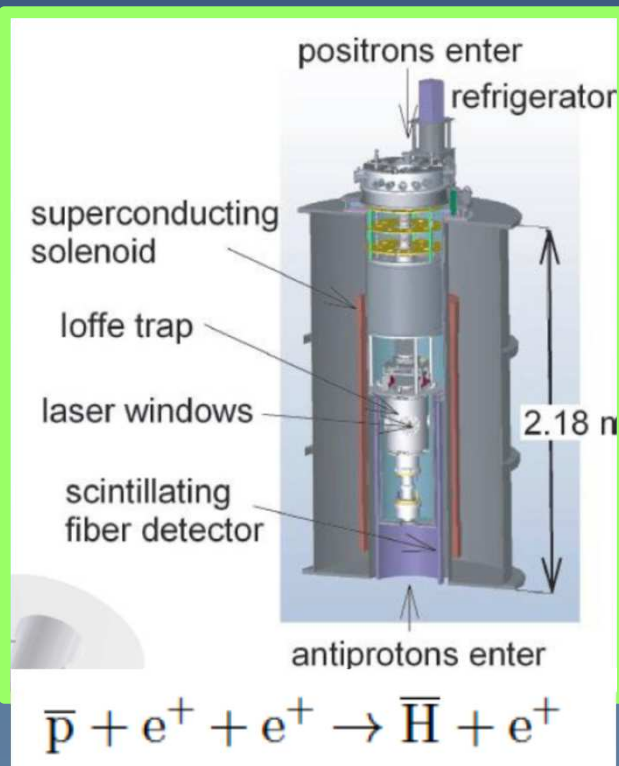
Nuclear Physics

-Ship Trap (GSI)
Z>92 mass measurement

-Isolde Trap (Cern)

- JYFL Trap (finland)

- ...



Particle Physics at Cern

M/q of P and Pbar (R=10⁻¹⁰)

-ATrap - ATrap2
Athena ...

-Anti Hydrogen
Production cooling..
-Decay measurement
-Atomic spectroscopy

End

- I) History and evolution of the spectrometers
- II) Magnetics spectro/separators with accelerator's beams
 - technical devices : quad, dipole
 - Beam optics concept
- III) Spectrometers without accelerator
 - 1 exemple for Astroparticle
 - Penning Traps

Back-up slides

Usefull relations : $B\rho$, E , W

Resolution & dispersion

More on Optical matrix : the matrix for a drift

Beam ellipsoid

Exemple 1: Big Rips in Tokyo (Riken)

Exemple 2: VaMOS in Caen (Ganil)

- Non linear effect in optical systems

Useful relations :

$$E_0 = mc^2 ; E = E_0 \gamma = mc^2 \gamma ; p = mc\beta\gamma ; cp = mc^2 \beta\gamma = E_0 \beta\gamma ; E^2 = E_0^2 + p^2 c^2$$

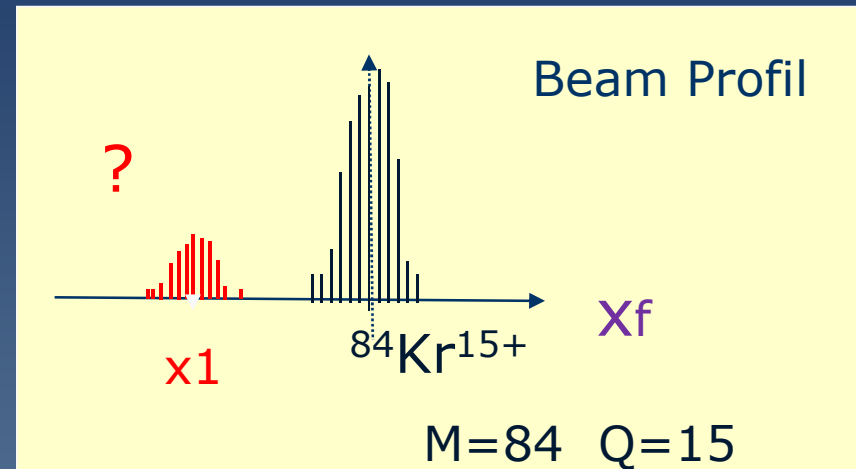
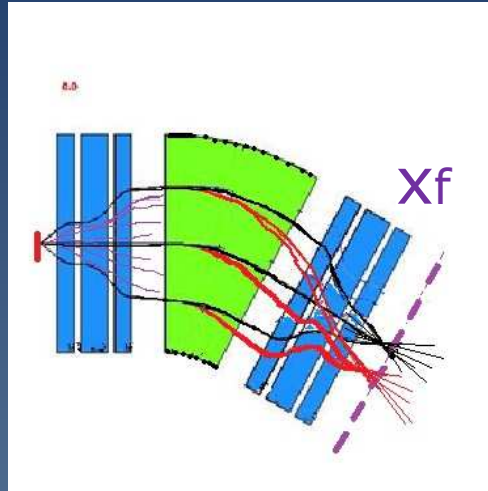
$$\beta\gamma = \frac{cp}{E_0} ; \gamma = (1 - \beta^2)^{-\frac{1}{2}} ; \beta^2 \gamma^2 = \gamma^2 - 1 ; W = E - E_0 ; \frac{mc\beta\gamma}{q} = B\rho .$$

	β	γ	W	cp
β	β	$\frac{\sqrt{\gamma^2 - 1}}{\gamma}$	$\frac{\sqrt{(1 + W/E_0)^2 - 1}}{1 + W/E_0}$	$\frac{cp/(mc^2)}{\sqrt{1 + [cp/(mc^2)]^2}}$
γ	$\frac{1}{\sqrt{1 - \beta^2}}$	γ	$1 + W/E_0$	$\sqrt{1 + \left(\frac{cp}{mc^2}\right)^2}$
W	$\left(\frac{1}{\sqrt{1 - \beta^2}} - 1\right)E_0$	$E_0(\gamma - 1)$	W	$mc^2 \left[\sqrt{1 + \left(\frac{cp}{mc^2}\right)^2} - 1 \right]$
cp	$mc^2 \frac{\beta}{\sqrt{1 - \beta^2}}$	$E_0(\gamma^2 - 1)^{1/2}$	$[W(2E_0 + W)]^{1/2}$	cp

Magnetic Spectrometer :

A tool for identification

Suppose 2 ions beams



-Field measurement B

$$B_{p0} = B_{dipole} * R_{dipole}$$

-Position measurement ($X_f = X_1$)

$$\delta = (B_{p1} - B_{p0}) / B_{p0} = X_1 / R_{16}$$

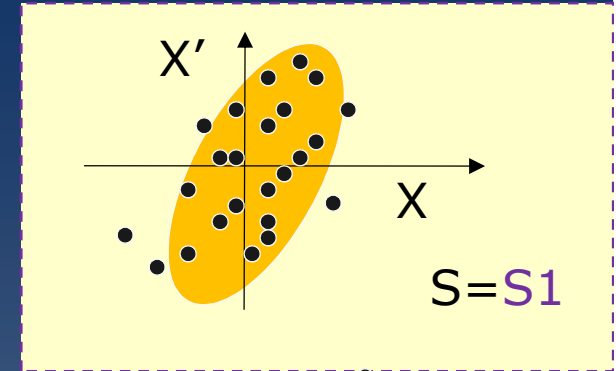
$$B_{p1} = B_{p0} (1 + X_1 / R_{16})$$

If same velocity v $M_1/Q_1 \approx M_0/Q_0 (1 + X_1 / R_{16})$

The **beam** : N particles in a 6D ellipsoid

$$\sigma_x^2 = \sigma_{xx} = \sigma_{11} = \frac{1}{N} \sum_{\alpha=1, \dots, N} (x_\alpha - \bar{x}) \cdot (x_\alpha - \bar{x})$$

$$\sigma_{xx'} = \sigma_{12} = \frac{1}{N} \sum_{\alpha=1, \dots, N} (x_\alpha - \bar{x}) \cdot (x'_\alpha - \bar{x}')$$



1) σ_{ij} is a statistical definition of the beam

2) An optical code

Computes σ_{Final} with the R matrix at the end of the spectrometer

$$\sigma_{final} = R^T \cdot \sigma \cdot R$$



Done by simulation code

R Matrix allows the simulation

- a) -of the beam size $\sigma(s)$
- b) -of one trajectory $Z(s)$

Beam emittance : (# optical quality)

The **emittance** is a **volume of phase space** occupied by a beam

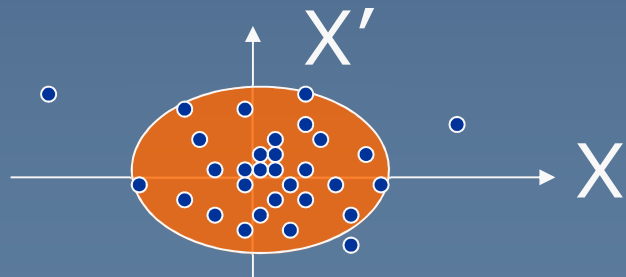
6 Dimensions

For practical reasons we use the subspace measurement **(x,x')** & **(y,y')**

Horizontal Emittance : area in (x,x')

Vertical Emittance : area in (y,y')

Longitudinal Emittance : area in (energy ,time)



$$\varepsilon_{rms} = 4(\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2)^{1/2}$$

ε = area of the ellipse ,which

correspond to x% particles

94

Liouville theoreme : emittance is conserved in a beam line..

More on Transport Matrices: Rmatrix for a straight section L (drift)

Particle Evolution in **drift** length between s_1 & s_2 :

$$x=x(s) \quad y=y(s) \text{ ???????}$$

$$x_2 = x_1 + \tan(\theta_1)(s_1 - s_2)$$

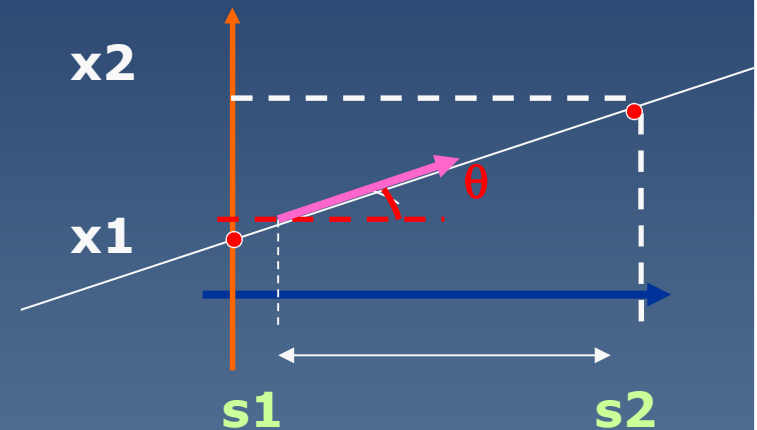
$$\theta_1 = \theta_2$$

$$y_2 = y_1 + \tan(\varphi_1)(s_1 - s_2)$$

$$\varphi_1 = \varphi_2$$

nota: $\tan(\theta_1) = dx_1/ds = x_1'$

and $(s_2 - s_1) = L$



$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

$$R_{d1} = \begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemple n°1: fragments separator @Riken(Japan)

E#300-500 MeV/A L=77m

6 dipoles magnets, 42 quadrupole magnet

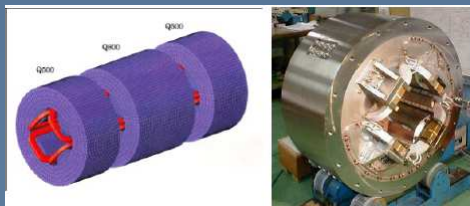
Suppression of the primary beam

(many dipoles, degrador selection)

Help the selection of very rare nuclei

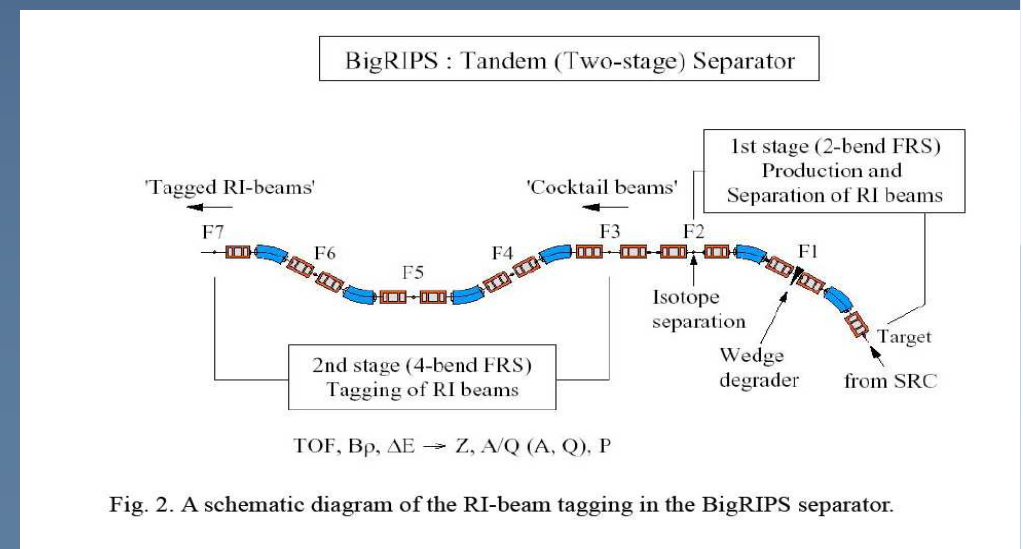
Selection of 4-5 nuclei

Identification (DE-TOF)



Superferric quads

B.Jacquot// Ganil

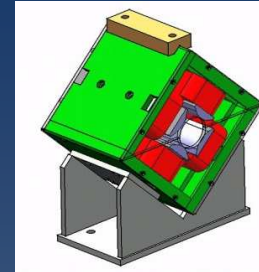
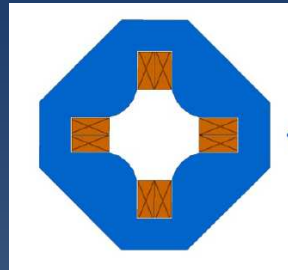


Quadrupole technology

1 : Normal conducting quad

hyperbolic pole (Fe)
coils (Cu)

$G \sim 10$ Tesla/m



Larger Aperture
or/and
Higher strength

2 : Superferric quad

hyperbolic pole (Fe)
coils (NbTi)

Higher Gradient , larger aperture
possible (A1900, BigRips, Synchro.)

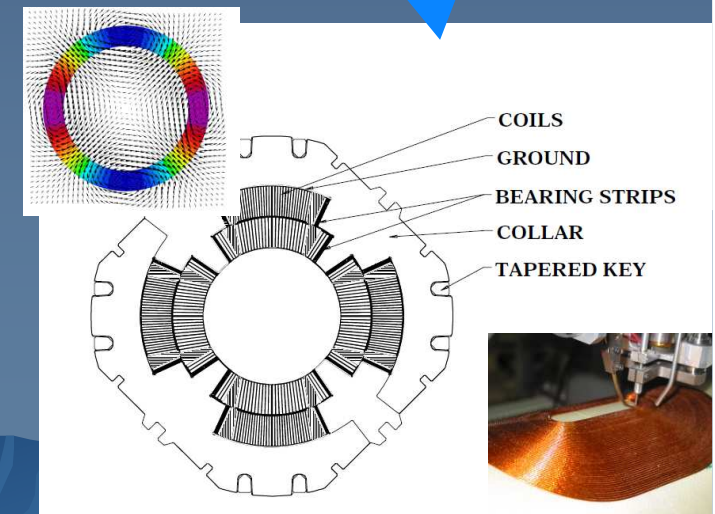
$G \sim 20-30$ Tesla/m



3 : Superconducting quad

No pole !!!!!
 $\cos(2\theta)$ coils (NbTi)

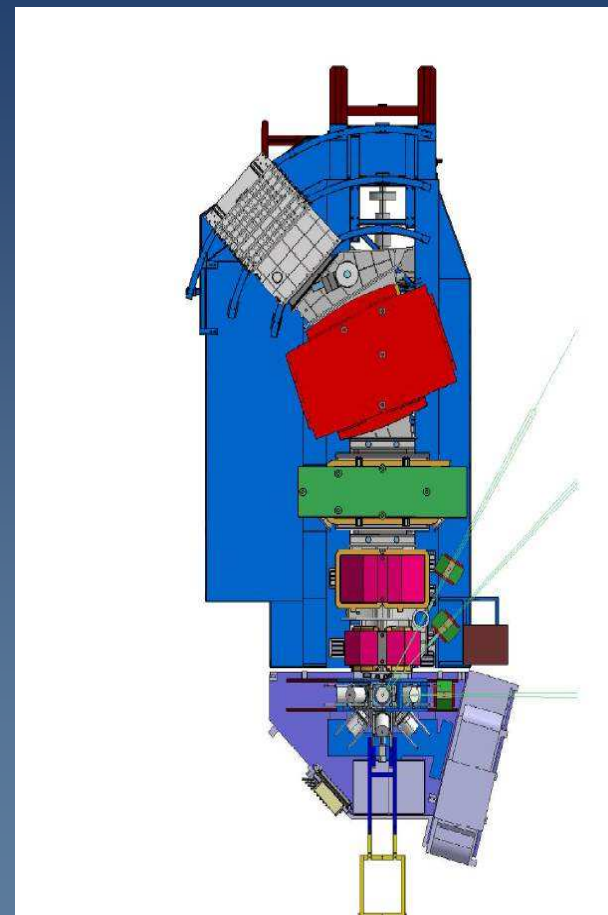
$G \sim 40-200$ Tesla/m (Cern LHC...)



Exemple n°2: VAMOS Spectrometer

L=8 meters, 1 dipole, rotative platform

Large angular Acceptance spectrometer : 70mstrd



Exemple n°2: VAMOS Spectrometer (Ganil)

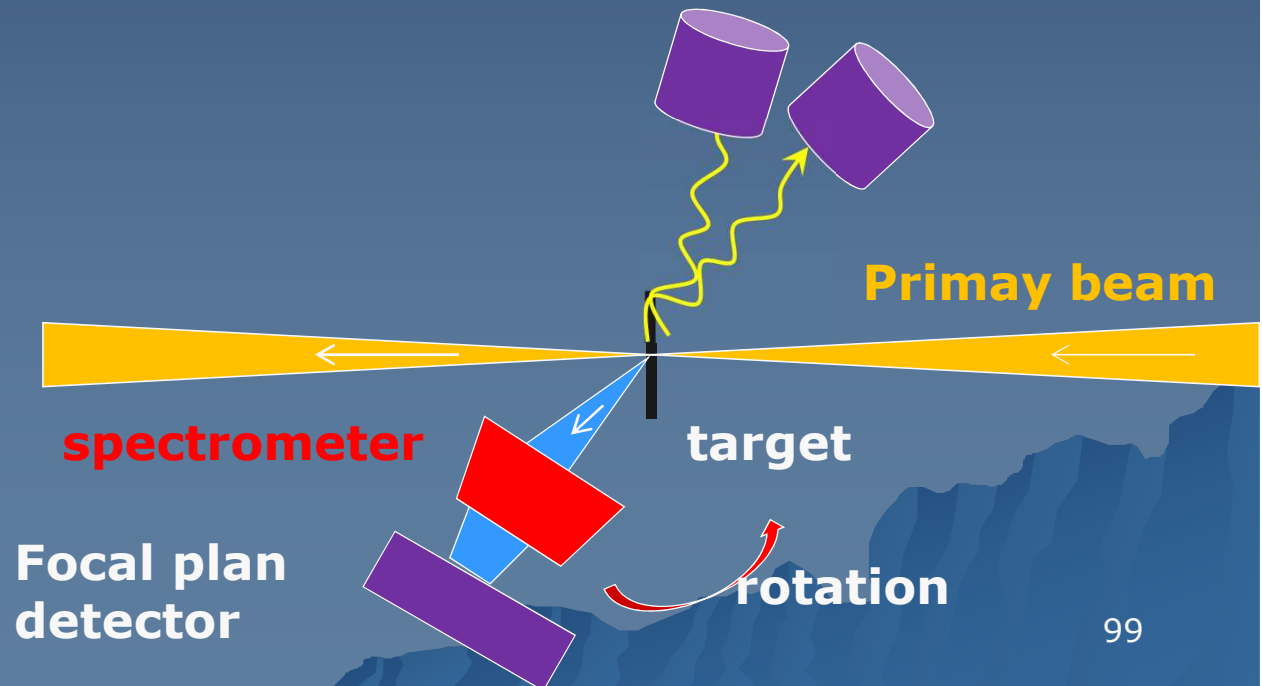
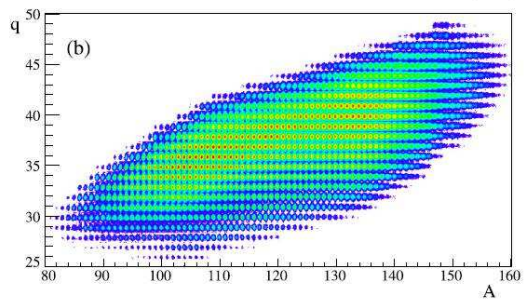
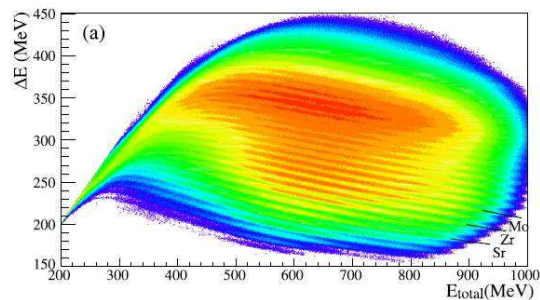


Suppression of the primary beam
(by rotation)

Selection of 20-300 nuclei

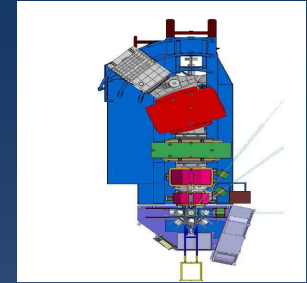
Help Identification (ΔE -TOF,
position and angle measurements)

300 fission fragments id.

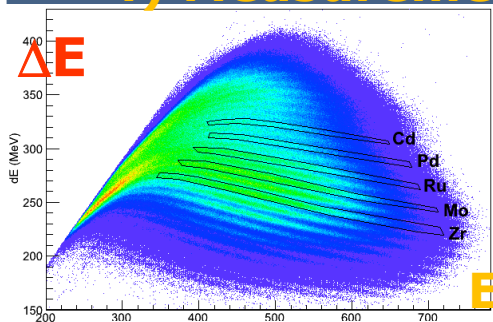


Example n°2: VAMOS Spectrometer

Particle identification Method (M, q, Z)



- 1) Measurement of the time of flight (TOF) => velocity
- 2) Measurement of the position x_{focal} after the spectrometer
=> $B\rho = B \times R_{dipole} (1 + x/R_{16} + \dots)$
- 3) Measurement of the energy loss ΔE in a thin detector (Ionization Chamber)
- 4) Measurement of residual energy E_r ($E_{kinetic} = (\gamma - 1)M c^2$)



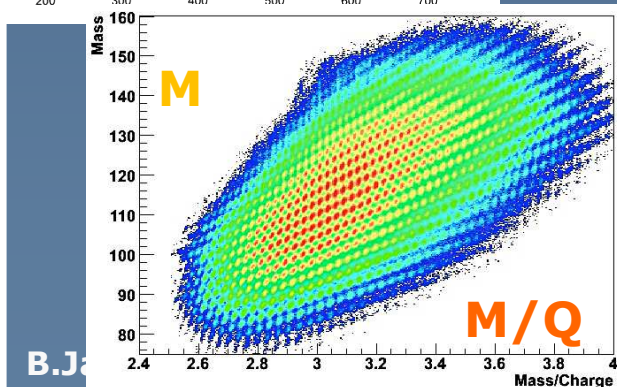
v
 M/q
 Z
 M_1

$$v = T_{flight} / L_0$$

$$M/q = B\rho / \gamma v$$

$$Z \# k \Delta E \dots$$

$$M_1 = (E_r + \Delta E) / [c^2 (\gamma - 1)]$$



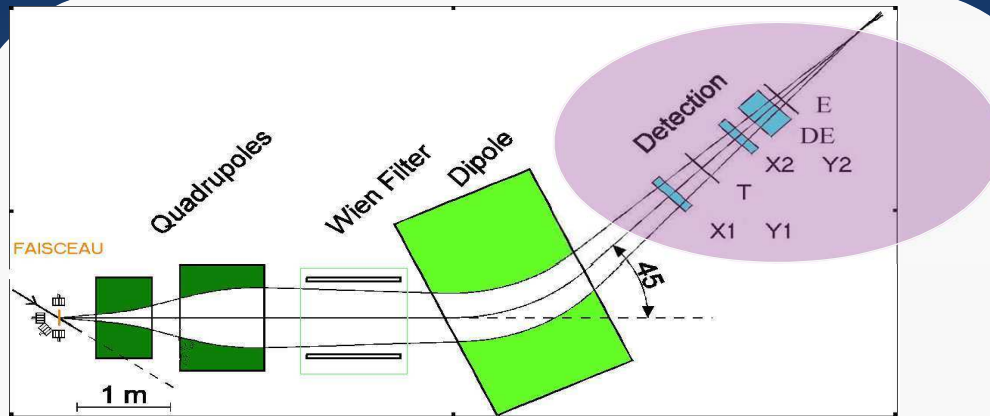
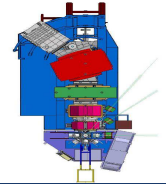
finally

$$Q = M_1 / [M/q]$$

$$M = [M/q] \cdot Q$$

$$Z \# k(E) \Delta E$$

Example n°2: VAMOS Identification



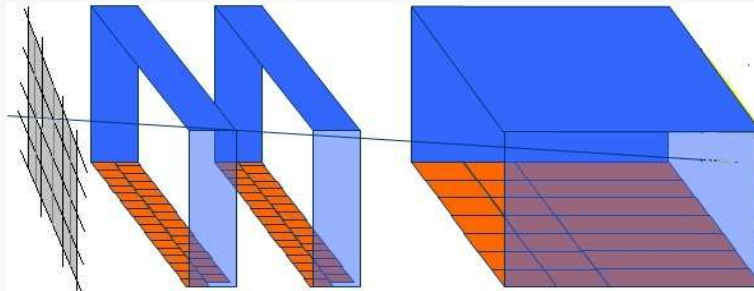
In the focal plane, 7 quantities are measured :
T, x1, y1, x2, y2, ΔE, E

T : Multi Wire PPAC

x1, y1
x2, y2 :

$$x' = (x_1 - x_2) / d = \tan(\theta)$$

$$y' = (y_1 - y_2) / d = \tan(\phi)$$



MWPPAC

(Tof)

Drift Ch.

(X, X', Y, Y')

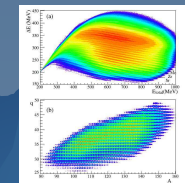
Ionis. Ch.

(ΔE, E)

ΔE, E : ionisation CHAMBER

$$B\rho = B\rho_0 (1 + x/R_{16} + a x'^2 + b x^2 + c x^3 + \dots)$$

Equation is non-linear in x, x', y, y' (Aberrations)



Non linear effects in optical system

1rst order

$$\vec{Z}_2 = R \cdot \vec{Z}_1 + \dots \varepsilon$$

for large angle, large B_p
deviation 2nd order, third order
is required.

$$Z_{2i} = \sum_{j=1}^6 R_{ij} \cdot Z_{1j} + \sum_{k=1}^6 \sum_{j=1}^6 T_{ijk} Z_{1j} \cdot Z_{1k} + \dots$$

1rst order

2nd order

Linear Approximation holds for
small angle, small B_p
deviation... (#30mrad, $\delta < 2\%$)

$$Z_1 = (x, x', y, y', l, \delta)_1$$

Effects of second order :

- Inclination of focal plane
- the Focusing strenght of quads is b_p dependant
- Large angle particles are not well focused

Non linearities (ABERRATIONS) come

- with large acceptance (large x' and large δ)
- but also, with field defects in quads and dipoles

Non linear effects in optical system

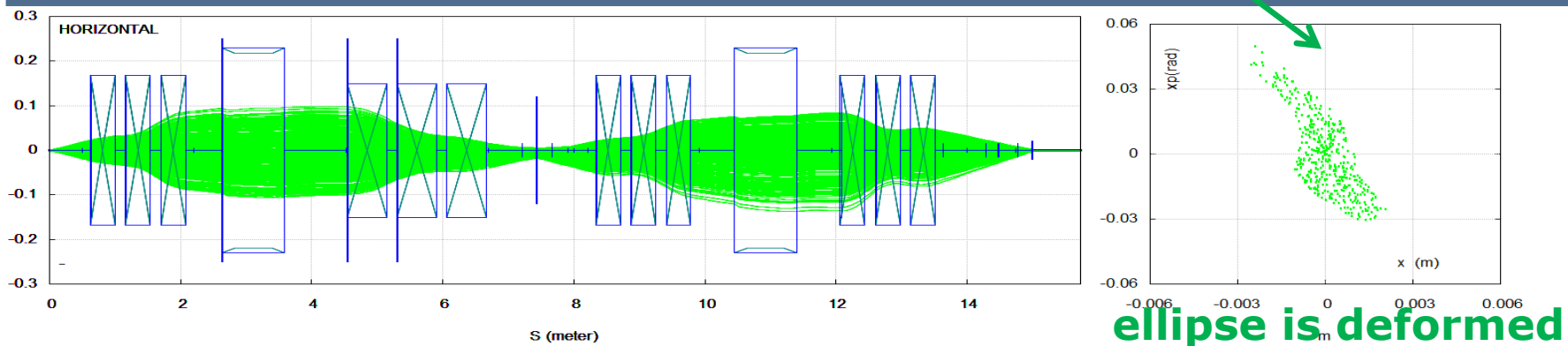
Beam optics is linear when $x < 5\text{cm}$
 $x' < 30\text{mrad}$
 $\delta < 2\%$

Beam is a nice ellipse in phase space, R matrix is sufficient

If $|X'| > 30\text{mrad}$ or $|\delta| > 2\%$

Beam are not well represented by an ellipse

R matrix is **not sufficient** for the calculation
(field maps + tracking with « Runge kutta » simulation needed)

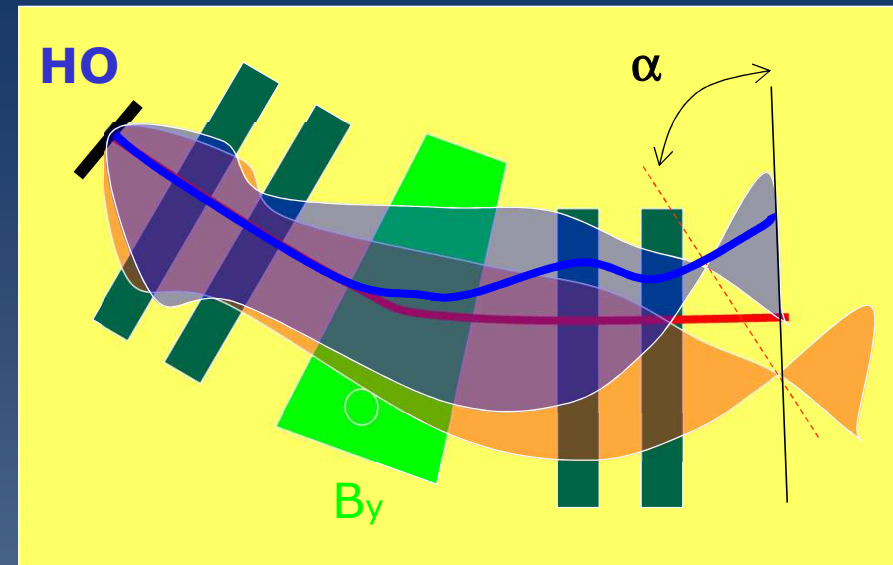


Non linear effects in optical system

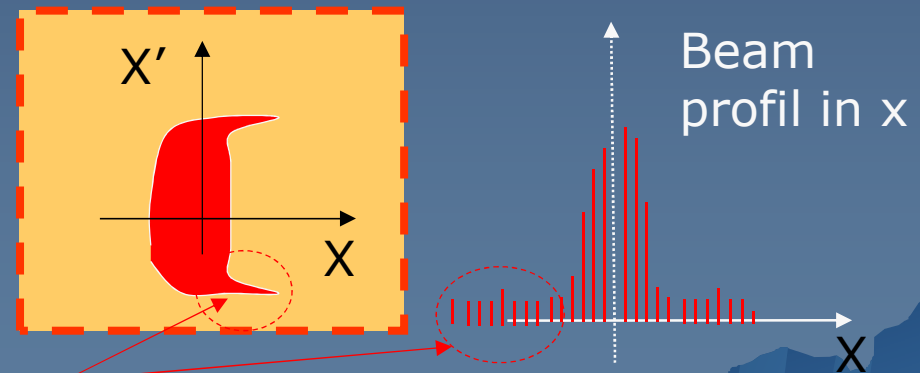
Ex1: Inclination α of the focal
in a spectrometer

$$\text{tg}(\alpha) = R_{16} / T_{126} \cdot R_{11}$$

- Choice of the **dipole Angle**
- Magnetic sextupole has to be used for correction



Ex2: distorsion of beam ellipse
In phase space
Inducing Distribution wings



Optical aberrations (non linearities)